VENDOR MANAGED INVENTORY PROGRAMS AND THEIR EFFECT ON
SUPPLY CHAIN PERFORMANCE

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Vendor Managed Inventory (VMI) programs are one popular method of placing a supply chain under centralized control. Under VMI, the customer supplies the vendor with inventory information and the vendor uses this information to manage the customer’s inventory. In this respect, the customer authorizes the vendor to make all inventory replenishment decisions. This dissertation investigates how VMI enhances a manufacturer’s production timing and results in improved levels of service and inventory.

In the first study, we compare three replenishment strategies in a two-echelon serial supply chain. These strategies include Make-to-Order, Make-to-Stock, and Vendor Managed Inventory. To compare these strategies, we develop several probability models that we use to calculate the expected fill rate, average customer inventory, and average manufacturer inventory. The second study determines the benefit of VMI in a two-echelon arborescent supply chain. Assuming that each customer orders a unique product, most of VMI derives much of its benefit from improved production timing. To this end, three reordering policies are investigated, including echelon and installation stock policies. In addition, we propose a heuristic policy which calculates a power function of the customers’ inventory positions and initiates production once the result of this function reaches a threshold value. Numerical analysis is used in both studies to determine the impact of several key environmental variables.
# TABLE OF CONTENTS

1. VENDOR MANAGED INVENTORY SYSTEMS .................................................................1
   1.1 Introduction ............................................................................................................1
   1.2 Benefits of Central Control of the Supply Chain ..................................................2
   1.3 Limitations of VMI ..............................................................................................3
   1.4 Research Scope ....................................................................................................5
   1.5 Research Contributions ......................................................................................7

2. LITERATURE REVIEW .............................................................................................9
   2.1 Value of Information Sharing and Centralized Control in a Serial Supply Chain ...10
   2.2 Value of Information Sharing and Centralized Control in Arborescent Supply Chains ...18
   2.3 Empirical Research ............................................................................................26
      2.3.1 Case / Survey Research .................................................................................26
      2.3.2 Experimental Research ................................................................................28
   2.4 Stochastic Economic Lot Scheduling Problem ....................................................30
   2.5 Comparison of Dissertation to Extant Literature .................................................35

3. VENDOR MANAGED INVENTORY IN A SERIAL SUPPLY CHAIN .........................38
   3.1 Introduction ............................................................................................................38
   3.2 Supply Chain, Replenishment Models, and Performance Measures Description ...40
   3.3 Analytic Development ..........................................................................................44
      3.3.1 Expected Number of Replenishments between Renewals ............................45
      3.3.2 MTO and MTS Models ................................................................................47
      3.3.3 Vendor Managed Inventory Systems ..........................................................51
   3.4 Numerical Analysis ...............................................................................................56
      3.4.1 Factor Levels ................................................................................................56
      3.4.2 Numeric Results ............................................................................................57
   3.5 Serial Supply Chain Simulation ............................................................................61
      3.5.1 Simulation Parameters and Factors ..............................................................65
      3.5.2 Simulation Results .......................................................................................66
   3.6 Results Discussion ...............................................................................................80

4. VMI PRODUCTION BENEFIT IN AN ARBORESCENT SUPPLY CHAIN ...............83
   4.1 Introduction ............................................................................................................83
   4.2 Supply Chain Description .....................................................................................85
   4.3 Production Models ...............................................................................................88
   4.4 Arborescent Supply Chain Simulation .................................................................93
      4.4.1 Factor Levels ................................................................................................94
      4.4.2 Simulation Verification ..................................................................................96
   4.5 Simulation Results ...............................................................................................100
      4.5.1 Factor Effects on Average Inventory ............................................................100
      4.5.2 Comparison of Models ...............................................................................102
   4.6 Results Discussion ...............................................................................................110

5. CONCLUSIONS AND FUTURE WORK ..................................................................112

APPENDICES .............................................................................................................115
   Appendix A. Simulation Code for Study 1 .................................................................115
   Appendix B – Simulation Code for Study 2 .............................................................124

REFERENCES ..............................................................................................................133
1. VENDOR MANAGED INVENTORY SYSTEMS

1.1 Introduction

Supply chain management (SCM) is utilized by companies to improve their competitive position in the marketplace. In contrast to a traditional supply chain where companies work independently, SCM involves close coordination and synchronization of information and material flows. While there are a variety of tools for synchronizing the supply chain, this dissertation focuses on one popular strategy - Vendor Managed Inventory (VMI) programs. As information sharing and collaboration becomes more predominant in industry, VMI is becoming a major development in supply chain management (Mishra and Raghunathan, 2004).

The relationship between supply chain members participating in a VMI program differs markedly from those maintaining a traditional supply chain. In traditional supply chains, information sharing is minimal and there is no collaboration between supply chain members. Consequently, the relationship between a vendor and its customers is limited to the vendor filling customer orders (Gavirneni 2001). Under VMI, however, the customer supplies the vendor with inventory information and the vendor uses this information to manage the customer’s inventory. In this respect, the customer authorizes the vendor to make all inventory replenishment decisions, including the timing of shipments and the replenishment quantity.

There have been a number of studies in the popular press concerning VMI. A study of consumer products manufacturers and distributors by Prescient found 75% of respondents were currently engaged in a VMI program with an average of six retail partners. The same study notes that VMI was experiencing a resurgence of interest in the
consumer products market (Supply & Demand Chain Executive 2003). In a survey of manufacturers and retailers in the food and beverage, electronics, and chemical industries, it was discovered that half of respondents were utilizing VMI (Daugherty et al. 1999). Research by AMR found that companies who had implemented VMI reduced lead time by 50%, inventory levels by 20% to 70%, and improved the in-stock rate by 1% to 12%.

1.2 Benefits of Central Control of the Supply Chain

The extant literature has established that a centrally controlled supply chain offers several benefits to supply chain members. Most significant among these benefits are reduced inventories and improved service levels. Previous studies have investigated several mechanisms through which supply chains obtain these benefits. For instance, when a vendor has replenishment responsibilities for several customers who are in close proximity, the vendor may consolidate the shipments to these customers. Similarly, if a single customer orders several products from the vendor, the products may be consolidated into a single shipment rather than shipping each product separately. The coordination of orders is a second major benefit of centrally controlled supply chains. In this case, a supplier, such as a central warehouse, times the receipt of its inbound shipments to coincide with the time it sends outbound shipments to its customers. In this way, the supplier reduces the inventory it holds on site.

When the vendor replenishes several customers, another available benefit is the allocation of on-hand inventory amongst the customers. Here, rather than sending the customers a fixed quantity at each shipment, the vendor varies the quantities for every
customer at each shipment. For example, rather than receiving a constant shipment of 100 units, a customer may receive 78 units one period and 103 the next. The quantity shipped to a particular customer depends on that customer’s inventory level and the inventory levels of the other customers the vendor replenishes. The vendor makes the allocation decision by considering each customer’s inventory level and the amount of inventory that is available for allocation. For this study, the objective when determining shipment quantities is to equalize the customers’ service levels. By assigning inventory in this manner, the vendor maximizes the average supply chain service level achievable using the available inventory.

The reduction of the bullwhip effect, a phenomena typified by increasing order fluctuations from lower to higher echelons in the supply chain, is a final benefit of coordinated supply chains. The bullwhip effect causes inefficient use of such resources as inventory and manufacturing capacity (Lee, So, & Tang 2000). These resources are affected because increased order variability necessitates either additional manufacturing capacity to provide reasonable response times or additional inventory to guard against the largest orders. When information is shared in a coordinated supply chain, some causes of the bullwhip effect are eliminated, thereby decreasing supply chain uncertainty and increasing the efficient utilization of resources (Lee et al. 2000).

1.3 Limitations of VMI

Despite the reported benefits of a centrally controlled supply chain, other studies in practitioner-oriented journals have shown moving to a VMI program does not necessarily benefit all supply chain members (Cooke 1998). For instance, a study in the
electronics industry found that 62% of customers were able to lower their costs by moving to VMI, but only 11% of vendors could make the same claim. The same study found that 49% of vendors had increased costs associated with moving to a VMI program (Baljko 2003). It has also been noted that VMI is not always implemented to its full potential due to manufacturing planning systems not using consumer demand information (Schenck and McInerney 1998). Other practical issues preventing a successful VMI would include non-standardized product identification across the supply chain and difficulties integrating information systems (Kaipia et al. 2002).

Another reason for the discrepancy between academic and practitioner accounts of VMI’s benefit is a critical difference between assumption and reality. In practice, many of the VMI benefits are not obtainable due to the specific environment in which it is implemented. Take for example Kimball Electronics, a large electronics manufacturer headquartered in Jasper, Indiana, visited as part of this research. Kimball acts as a Tier 2 supplier in the automotive industry and participates in a VMI system with several of its Tier 1 customers. Unlike most supply chains studied in the literature, Kimball’s products are designed specifically to customer specifications. Consequently, one of VMI’s primary benefits, the allocation of inventory amongst customers, is eliminated. In addition, it may not be possible to consolidate shipments to Kimball’s customers, further reducing the benefit of a VMI program. A key benefit VMI offers Kimball, however, is the ability to allocate manufacturing capacity across different products. The benefit of allocating capacity in such a manner is investigated in this dissertation.
1.4 Research Scope

Since many of VMI’s benefits may not be applicable in all supply chains, a primary objective of this research is to study VMI programs in situations where they derive their benefit exclusively from improved production and shipment timing. This objective is addressed in both serial (Study 1) and arborescent (Study 2) supply chains. To control production and shipment timing, an implementation of VMI which utilizes two parameters, \( s_1 \) and \( s_2 \), is introduced. The first parameter, \( s_1 \), is the customer inventory level at which the manufacturer initiates production while the second parameter, \( s_2 \), is the customer inventory level at which a shipment is sent from the manufacturer. By timing production and shipments in this manner, the manufacturer is able to manipulate the inventory levels across the supply chain. As a result, depending on the holding costs at the manufacturer and the customer, VMI may provide a lower total supply chain cost than other replenishment strategies.

Cachon and Fisher (2000) note that the benefit of programs such as VMI differs across studies because of differences in assumptions and environmental factors. Consequently, another objective of this research is to quantify the impact that several environmental factors have on VMI’s performance. For Study 1, these factors include the shipping lead time, coefficient of demand variation, the capacity utilization, lot sizes, and fill rate. Study 2 investigates these factors’ when multiple nonidentical products compete for capacity in an arborescent supply chain having.

Sahin and Robinson (2005) note that the supply chain literature focuses exclusively on environments where the vendor follows a Make-to-Stock (MTS) replenishment strategy. This is an important oversight since many manufacturers,
including Kimball Electronics, operate using a Make-to-Order (MTO) strategy. Thus, the benefit a program such as VMI offers to a manufacturer currently following an MTO strategy is not well established. Consequently, the final objective of this dissertation is to investigate how beneficial VMI is relative to an MTO replenishment strategy.

To meet these objectives, the first study compares the performance of VMI, MTO, and MTS replenishment strategies in a two-echelon serial supply chain with finite production capacity. With an MTO strategy, the manufacturer produces in response to a customer order, whereas the manufacturer produces in anticipation of orders with an MTS strategy. Under VMI, however, the customer gives the manufacturer authority to select both a production and shipping signal based on the customer’s inventory level. For each replenishment strategy, we develop a probability model to calculate the expected fill rate, average customer inventory, and average manufacturer inventory. We then calculate the average customer and manufacturer inventory necessary for a target fill rate for each of the three strategies using a numeric analysis.

The second study investigates the benefit of VMI in an arborescent supply chain. Here the benefit of VMI directly relates to the manufacturer’s ability to utilize real time customer inventory information to make production decisions that benefits all customers. In this study, each customer orders a unique product and the manufacturer can produce only one product at a given time. Additionally, the customers in this supply chain need not be identical. We compare three models that correspond to different reordering strategies, including one based on individual customer inventory levels, another based on aggregated customer inventory, and finally a heuristic reorder policy developed in the study. Under the heuristic policy, the manufacturer checks the customer’s inventory.
levels periodically. In each period, the manufacturer calculates the expected fill rate for each product’s next replenishment assuming it immediately begins production and based on the product’s current inventory level. The manufacturer also calculates a product’s fill rate assuming this product cannot be replenished until production for the other product is completed. The heuristic initiates production once the expected fill rate for this one replenishment reaches a threshold value.

1.5 Research Contributions

A large body of literature investigates multi-echelon distribution systems. The study of multi-echelon supply chains where a manufacturer occupies the higher echelon, however, comprises a very small portion of this literature. In addition, much of the literature has focused on the distribution benefits of centrally controlled supply chains, such as shipment consolidation and allocation of inventory. This research also investigates the benefit of VMI, which results purely from improved production timing. The proposed research contributes to the literature in several ways.

- Utilizes two parameters, \( s_1 \) and \( s_2 \) to introduce a VMI implementation similar in structure to a standard reorder point inventory policy and compares it to MTS and MTO policies.
- Use renewal theory to analytically calculate the customer’s safety stock, the manufacturer inventory, and fill rate for VMI, MTS, and MTO strategies.
- Determines the conditions under which VMI is most beneficial and the factors that contribute to this.
• Demonstrates that VMI programs enable the manufacturer to reduce total cost by allowing the manufacturer to select policies that create efficient levels of inventory across the supply chain.

• Investigates reorder policy choice in a supply chain having several similar, but unique, products.

• Introduces a heuristic reorder policy that minimizes inventory needed to achieve a long-run fill rate.

The VMI implementation used in these studies is unique in that it looks purely at the benefit derived from improved production and shipment timing. There is no inventory allocation and there are no lot sizing advantages as would occur if a customer received more frequent shipments. This study is also unique in that it considers an MTO replenishment strategy, whereas the literature traditionally compares policies such as VMI to MTS environments (Sahin and Robinson 2005).
2. LITERATURE REVIEW

The most relevant research to the studies included in this dissertation are those papers investigating the value of information sharing and centralized control. Two supply chain structures are typically modeled - serial and arborescent. A *serial* supply chain is a supply chain where each echelon replenishes only a single downstream stage. Serial supply chains are typically modeled as having just two echelons, although it is possible to investigate additional levels. In an *arborescent* supply chain, an echelon may replenish more than one downstream stage. The arborescent supply chain literature almost exclusively considers two echelons, whereby a single supplier or warehouse replenishes $n$ customers.

Our second study investigates a situation where several products are manufactured on a single machine. Hence, the Stochastic Economic Lot Scheduling (SLSP) literature is also of interest. This literature has at its base the Economic Lot Sizing Problem (ELSP) whereby a production sequence and lot sizes are determined for multiple products having deterministic demand. SLSP extends the ESLP by considering stochastic demand and by allowing backorders.

The benefit of VMI initiatives have also been studied using survey and case research methodologies. While these papers are not as pertinent to our work, they are nevertheless included in the literature review since they deal specifically with VMI. Sahin and Robinson (2002) suggested several literature classification schemes, and using this as a basis, the literature review is structured in the following manner:
1. Value of Information sharing and centralized control in serial supply chains

2. Value of Information sharing and centralized control in arborescent supply chains

3. Empirical VMI research
   3.1. Case / Survey
   3.2. Experimental

4. Stochastic Economic Lot Scheduling problem

2.1 Value of Information Sharing and Centralized Control in a Serial Supply Chain

Information sharing occurs when one supply chain party provides the other supply chain members with what had previously been private or internal information. Often, information flows from a downstream party (e.g. a retailer) to an upstream party (e.g. a vendor) although there are instances where a vendor can furnish a retailer with information such as its own inventory levels or its shipment receipt dates.

In general, the papers in this section compare the performance of a model having information sharing to a model with no sharing. In this way, the value of using information is established. In models with no information sharing, it is typically assumed that the vendor is only aware of the orders made by its customers. In models with information sharing, many different forms of information are shared, including inventory positions, end-customer (consumer) demand in each period, the customer’s demand distribution, and the inventory policy being used by a customer, including the type of policy, order quantities, and reorder points.
In addition to information sharing, a supply chain may also be placed under centralized control. Centralized control implies that a single entity makes all decisions with the purpose of maximizing the supply chain profit. Much of the research in this section studies the additional benefit of central control of a supply chain beyond the benefit of information sharing alone. The research on serial supply chains tends to investigate the form of contracts between the vendor and customer whereas the research on arborescent supply chains studies the benefits of risk pooling and allocating stock.

The seminal work on serial supply chains is by Clark and Scarf (1960) who investigated periodic review base-stock policies. Demand only occurs at the customer in the lowest stage, there are no setup costs except possibly the highest stage, and excess demand is backordered. The authors use the concept of echelon stock in determining optimal base-stock levels. For a given stage, Clark and Scarf define its echelon stock as the inventory on-hand at this stage plus on-hand and in-transit inventory at all lower stages. The authors demonstrate that each stage’s optimal purchase quantity is a function of the inventory in-transit to this stage in addition to its echelon stock. To calculate the optimal purchase quantity, the authors first define the natural one-period costs of a stage as the costs associated with the echelon inventory and in-transit inventory at that stage. The authors show that the optimal base-stock level for a stage can be calculated based on the natural one-period costs and using the same method as for a typical single installation. However, in addition to the natural costs, a cost which encompasses the effect that a shortage of inventory at a higher stage has on the lower stage must be included when calculating optimal base-stock levels.
Chen (1998) compares the performance between inventory control policies using echelon stock information and installation stock information in a multi-echelon serial system having N stages. Each stage in the system orders according to an \((R, nQ)\) inventory policy. Under an \((R, nQ)\) policy, there is a base order quantity, \(Q_i\), for each stage. When stage \(i\) \((i = 1, \ldots, N)\) has its inventory reach its reorder level, \(R_i\), it places an order for an integer multiple, \(n_i\), of the downstream stage’s base quantity. Demand is backordered at each stage but is penalized only at stage 1. The inventory policy which utilizes echelon stock information requires all stages to have access to their echelon inventory information and the stage 1 demand information, whereas policies using installation stock information need no such information. The author assumes that the base quantity, \(Q\), is exogenous and thus only needs to find the reorder point, \(R\).

Chen uses a computational study to determine the affect of moving from inventory policies based only on local information (installation stock) to policies based on information sharing (echelon stock). He finds that information becomes more valuable as batch sizes increase. This is explained by noting that installation stock policies, because they do not share information, are subject to the bullwhip effect and increasing demand distortion. Consequently, as batch sizes increase, the distortion becomes worse and information thus becomes more valuable. Chen also finds that information becomes more valuable as the number of stages in the supply chain increases and as the lead time increases. These two factors essentially increase the length of the supply chain and thereby increase distortion of demand information. A surprising result is that increased demand variability leads to decreased information value. This is explained by noting that added variability may cause the total cost of both the echelon
and installation stock models to increase to the point where the percentage difference between the two decreases.

Gavirneni, Kapuscinski, and Tayur (1999) investigate a two-echelon serial system consisting of a single retailer who follows an (s, S) inventory policy and orders from a single, capacitated manufacturer who follows a modified order-up-to policy. Each period begins with both the retailer and the manufacturer receiving shipments sent in the previous period. Consequently, the shipment lead time for both parties is equal to zero. After the shipment arrives, the manufacturer chooses a production quantity followed by the retailer experiencing consumer demand. The retailer then places an order with the manufacturer if its inventory position is below s. In situations where the manufacturer cannot completely fill the retailer’s order using on-hand inventory, the manufacturer can purchase material from an alternate source to fill the retailer’s order in its entirety. This assumption of having a readily available alternate source may not be practical in many situations, especially for a production environment.

Three levels of information sharing are examined: (1) no sharing, (2) sharing of the retailer’s ordering policy, i.e. the type of policy, the reorder point, and the order-up-to level, and (3) sharing of point-of-sale / inventory level information. The authors discuss that the manufacturer’s optimal order-up-to level changes each period when there is information sharing. The actual order-up-to level is calculated using infinitesimal perturbation analysis.

The authors use numerical analysis to compare the three models. The comparison between the model where there is no information sharing and the model where the retailer shares its ordering policy information reveals that system performance improves by 50%
on average. Results also indicate that as the manufacturer’s capacity increases, the benefit of sharing order policy information becomes greater because the supplier gains flexibility in scheduling production. Information is found to be most valuable for moderate levels of demand variance and when the retailer orders large batches. Sharing point-of-sale information makes improvements ranging from 1% to 35% compared to sharing only the order policy information. The relationship between the value of information and capacity, demand variance, and retailer order quantity hold true when point-of-sale and inventory information are shared.

Lee, So, and Tang (2000) consider a two-echelon serial supply chain consisting of a single retailer and a single manufacturer with autocorrelated demand. Both parties use a periodic review system to replenish their stock. The results of this paper demonstrate that information sharing improves the manufacturer’s inventory levels, and the value of information sharing increases with increased demand autocorrelation and increased lead times.

Fry, Kapuscinski, and Olsen (2001) investigate the benefits offered by VMI systems over traditional retailer-managed inventory systems with information sharing in a two-echelon serial supply chain. The manufacturer (the vendor) is modeled as producing only once every few periods, but its production quantity is unlimited in that period.

The authors propose a (z, Z) inventory policy where z and Z denote the minimum and maximum inventory levels and are set by the retailer. The costs associated with this policy are the holding costs at both the supplier and the retailer. The retailer is also charged a backorder cost in case of a stockout, and if the supplier cannot completely fill a retailer’s order, it is charged for acquiring stock from an outside source. Additionally,
the supplier is contractually obligated to pay the retailer penalties if, after end-customer demand, the retailer’s inventory is either below $z$ or greater than $Z$.

It is found that the performance of the $(z, Z)$ contract does not always surpass that of the retailer-managed inventory system. This difference in performance can be attributed to the penalty costs associated with either surpassing the retailer-dictated minimum and maximum inventory levels. As such, the authors propose several guidelines for calculating these contractual penalty costs. While the proposed guidelines do not guarantee the optimal policy, they surpass the performance of a retailer-managed inventory system.

Results indicate that the benefit of using VMI increases as the cost of the supplier acquiring stock from an outside source increases, with the percent improvement in cost ranging from 8.25% to 23.05%. Other results show that the benefit of VMI increases as the variance in customer demand increases, with the improvement for moving to VMI ranging from 10.04 to 15.59%.

Cachon (2004) investigates a situation where there is a risk neutral supplier selling products to a risk-neutral retailer over a single season. In this situation, he compares three types of pricing contract. The retailer and supplier may enter into a push contract, whereby the retailer commits to a single purchase quantity before the beginning of the season and where there are no further replenishments after that point. A pull contract is another possibility. Here, the retailer purchases product from the supplier one unit at a time or as demand is realized. This can also be said to represent a VMI contract whereby the product is on consignment. The final contract is an advanced purchase discount, whereby the retailer purchases a certain amount before the season but may also
make additional purchases as the season progresses. In this case, the price charged for the products purchased at the beginning of the season is lower than what is charged for the product as the season progresses. Despite the customer making multiple orders to the supplier, there is still a single production of the product before the season begins.

Cachon finds that the value of coordinating mechanisms such as buy-backs and revenue sharing is not as great as once thought. This is because these mechanisms are typically compared only against the push contracts, not against pull (or VMI) contracts, which provide better efficiency than the former.

Hariharan and Zipkin (1995) propose a class of inventory control policies which utilize advanced demand information from customers, i.e., customers inform the supplier what their demands are in advance of placing an order. Customer demands follow a Poisson arrival process with each customer ordering a single unit. Each order comes with a due date and delivery of the order cannot be made before this time. The authors show that their control policy, an order-base-stock policy, is optimal in the case lead times are constant. The order-base-stock policy is similar to a standard base-stock policy. However, rather than triggering orders when an actual customer demand arises, as is the case for a standard base-stock policy, orders are triggered at the supplier when the customer notifies the supplier of an order. Note that demand is backordered.

Choi, Dai, and Song (2004) explore how common service measures included in VMI contracts, such as fill rate, are inadequate to ensure the desired service level at the customer. The authors demonstrate that a contract containing both the supplier's stockout rate and expected backorders is needed to guarantee a given end-customer service level. To this end, the authors consider a two-echelon supply chain that consists
of a single supplier who provides material to a single capacitated manufacturer using an MTO policy. To replicate a manufacturer using an MTO policy, the manufacturer is modeled as using a base-stock policy whereby the base-stock level of finished goods is set equal to zero. Each period the manufacturer first sees customer demand and then produces a quantity that will maintain the finished goods inventory level at zero. In case the manufacturer is unable to satisfy all end-customer demand, either because of limited capacity or a lack of raw materials, that demand is backordered.

The supplier, who is also capacitated, also follows a periodic review, modified base-stock policy to control the inventory at the manufacturer’s site. The supplier in this case resupplies the manufacturer at the end of the period. Using such an inventory system, the supplier technically does not witness the end-customer demand; rather, the supplier has visibility only of its own inventory located at the manufacturer’s warehouse. So in cases where demand is greater than the manufacturer’s capacity, the supplier will not have accurate customer demand information. While our definition of VMI necessitates information sharing, the authors remark that giving the supplier the ability to see end-customer demand will not change the paper’s conclusion. Finally, the authors note that their results can be extended to a case where the manufacturer follows an MTS policy.
2.2 Value of Information Sharing and Centralized Control in Arborescent Supply Chains

Moinzadeh (2002) examines an arborescent supply chain where the retailers follow a (Q, R) inventory policy. In case the supplier does not have enough on-hand inventory to satisfy a retailer order, the supplier replenishes the retailer once the full order becomes available rather than sending a partial shipment. Although the actual transportation time between the manufacturer and the retailer is fixed, this leads to a stochastic replenishment time. In situations where the retailer stocks out, the demand is backordered. The supplier is replenished from an outside supplier after a constant lead time.

The author models a situation where the supplier has access to the retailers’ inventory and demand information. In view of this, the author develops an order policy for the supplier that attempts to coordinate the supplier’s outside replenishment to coincide with the retailer’s orders. This is implemented by the supplier identifying two variables, \( m \) and \( s \). The variable \( m \) identifies the number of batches of size Q (which is equal to the retailers order size) to hold initially while the variable \( s \) defines the amount above each retailer’s reorder point, R, that will trigger the supplier to order.

The results indicate that information is most valuable in systems where the supplier’s replenishment lead time is much longer than other lead times in the distribution system. Additionally, information is most beneficial when there are not a large number of retailers, when order quantities are neither very large nor very small, and when the ratio of holding cost between the retailers to the suppliers is neither very large nor very small.
Gavirneni (2001) studies an arborescent supply chain where a capacitated manufacturer, operating under an MTO strategy, replenishes identical retailers who order using order-up-to policies. In each period, the retailers first place their orders, which the supplier fills immediately using an allocation procedure. After receiving the shipments, the retailer satisfies the customer orders from the on-hand inventory. If the manufacturer is not capable of filling all orders, there is no penalty cost.

The first model investigated contains no information sharing other than the orders placed by the retailers. This model utilizes a lexicographic allocation policy in periods where the retailers' orders are greater than the manufacturer’s capacity. Under a lexicographic policy, the retailers are ranked in terms of their importance and orders are filled completely starting with the most important retailer and then moving down the ranking until the inventory is exhausted. Since the retailers are prioritized in this manner, each will use a unique order-up-to level.

In the second model, the manufacturer knows the retailers’ inventory positions and thus is able to use an improved allocation policy. Under this new policy, the retailer having the lowest inventory level has its level brought up to that of the retailer with the second lowest level of inventory. These two then have their inventory levels brought up to that of the retailer with the third lowest level of inventory. This sequence continues until the manufacturer’s capacity is depleted.

In the third model, a complete redistribution of inventory occurs at the retailers when they are replenished. It should be noted that a transshipment cost is not included in the model. As in the second model, the retailers are not given static replenishment
priorities and instead are replenished based on need. Consequently, all retailers in the second and third models use the same order-up-to level.

Results demonstrate that information becomes more valuable with decreasing capacity, contradicting earlier work such as Chen (1998) and Gavirneni et al. (1999). The disparity arises because in the current paper the manufacturer can allocate inventory which makes information very useful when capacity is limited. Other results indicate that information becomes more beneficial when the cost of stockouts is increased, the demand variance increases, and the number of retailers decreases. This pattern of results holds in comparisons between models 1 and 2 and between models 2 and 3. As expected, the benefit in moving from model 1 to 2 is greater than moving from model 2 to 3. In fact, the results show the benefit of moving from model 2 to model 3 is below 0.02%.

Cachon and Fisher (2000) investigate the value of sharing information relative to reducing lead times and increasing order frequency in a two-level arborescent supply chain. Both the supplier and the retailers use a periodic review (R, nQ) inventory policy to make their replenishment decisions. In case of a shortage on the part of the supplier, the supplier allocates inventory using the batch priority method.

In a numerical analysis, the authors find that information sharing reduces supply chain costs by 2.2% on average. Part of the improvement arises from the supplier improving the timing of its own ordering decisions. Performance is also improved because information sharing allows allocation decisions to be made using the retailers’ inventory information rather than using the retailers’ order quantities. In addition, the allocation decision is improved by changing the timing of the allocation decision. Under no information sharing, the allocation decision is made when the order is placed;
however, with information sharing, the decision can be delayed until an order is shipped. Thus, inventory risk is pooled over that difference in time.

The authors study the impact of reducing lead times and increasing order frequency. The results indicate the reducing the lead times by approximately half causes a reduction in system costs of approximately 21% on average. Also, increasing the order frequency by halving the batch sizes yields a 22% improvement in performance.

Cheung and Lee (2002) investigate the impact of shipment consolidation, replenishment coordination, and stock rebalancing on supply chain performance. The authors assume the negligible transportation time between retailers to allow shipment consolidation.

Two models are analyzed in which the supplier continuously monitors the retailers’ inventory levels. In the first, once the total demand across retailers reaches Q, a shipment is made from the supplier to the retailers. Here, the value of Q represents the capacity of a truck so that the supplier always ships full truckloads. Upon shipment receipt, each retailer’s inventory level is returned to its order-up-to level. Demand is backordered at the retailers in case of a stockout. The supplier follows a \((Q_0, R_0)\) policy when making its own resupply decisions from an external supplier. Under this policy, once the total demands from the retailers reaches \(R_0\) the supplier places an order for \(Q_0\) units. Thus, when the order is received, the supplier’s inventory level will return to \(Q_0 + R_0\). Note that both \(Q_0\) and \(R_0\) are integer multiples of \(Q\), which allows for the coordinated replenishments.

The second model builds on the first, although the supplier is now allowed to rebalance the retailers’ stock once it receives its own shipment. The authors derive upper
and lower bounds on this solution since it is too complex to analyze mathematically. To simplify the problem, they assume that the retailers are identical and that the supplier carries an ample amount of stock.

The performance of the two models is compared to a benchmark model in which the retailers follow a (Q, R) inventory policy. The authors prove that if Q is greater than 2, then total cost achieved by model 1 is always lower than that obtained using the benchmark policy. Furthermore, the upper bound on the cost of model 2 is found to be an improvement over model 1 using a numerical example. Using the same numerical example, the authors conclude that the value of shipment coordination is higher when the number of retailers increases. Similarly, the value of stock rebalancing increases as the number of retailers increases, although the marginal benefit of rebalancing decreases for each additional retailer.

Äxsater, Marklund, and Silver (2002) investigate centralized control of an arborescent supply chain. Each period begins with the warehouse deciding whether to place an order with its supplier. Subsequently, a shipment from a previous period is delivered to the warehouse. Following the receipt of this shipment, either a portion or all of the stock is allocated to the retailers. Next, a shipment from a previous period arrives at the retailers. Finally, demand occurs at the retailers.

The authors introduce two heuristics. The first, the virtual assignment ordering rule, determines the timing and quantity of warehouse replenishments from an outside supplier. The replenishment decision is made using the warehouse’s echelon inventory level assuming the stock is assigned to the individual retailers at the moment a replenishment order is made to the external supplier. The second heuristic, the two-step
allocation rule, is utilized each period. In the first step, the quantity of inventory to ship to the retailers and the quantity to retain at the warehouse is determined. The second step calculates the shipment size for each individual retailer.

These two heuristics are shown to surpass the Clark and Scarf (1960) balance rule in situations where the warehouse is replenished in large batches and the retailers differ in their demand characteristics or service levels. It should be noted that the virtual assignment rule results in higher inventory levels than required in Clark and Scarf (1960) because the timing of the allocation decisions are different. In Clark and Scarf (1960), the inventory is allocated once it is received at the warehouse, whereas in Axsater et al., inventory is allocated once the order is placed.

Aviv and Federgruen (1998) compare three levels of integration between a supplier and multiple, nonidentical retailers. The first level represents the traditional retailer-managed inventory scenario whereby the retailers place orders to the supplier at the beginning of each period using a base-stock inventory policy. In the second level, the retailers continue to place orders with the supplier. However, they now share inventory and demand information. The maximum integration takes place in the third level, representing VMI, whereby the supplier makes quantity and timing decisions for replenishing the retailers. By analyzing these three levels of integration, the authors are able to determine the extent to which VMI improves system performance over information sharing alone.

For the first two levels, the authors compare two allocation policies used when the vendor experiences a stockout. In the first policy, inventory is randomly allocated to the retailers. The second policy, however, allocates inventory based on each retailer’s mean
demand and on the size of their orders. The authors find that this second rule significantly improves the total supply chain costs in both the retailer-managed inventory setting and when information is shared with the supplier. In addition, when using the second inventory allocation rule, sharing information always improves supply chain cost, although the improvement ranges from only 0% to 5%, with an average of 2%. When comparing the performance of these two levels, the results indicate the value of information sharing increases as production capacity becomes more limiting.

When developing the VMI model, the authors consider four components. First, they set the replenishment schedule for each retailer. In this case, they use a constant replenishment cycle to make the problem of centralized inventory control less complex. This is followed by calculating the quantity the supplier should order at the beginning of the period. Next, the quantity to ship to the retailers at the beginning of each period is calculated. Finally, the allocation decision is made. The authors find that using VMI lowers the total supply chain cost by an average of 4.7% over the case where only information sharing is used. It is apparent from their results that VMI becomes more beneficial as capacity utilization increases.

Çetinkaya and Lee (2000) consider shipment consolidation in a VMI setting with multiple retailers. Given that the supplier replenishes its own stock using an (s, S) inventory policy, the objective of the paper is to calculate the optimal retailer shipment quantities and replenishment frequencies. The decisions involve balancing a transportation cost with a waiting cost. Here, a waiting cost is calculated as the difference in time between when a demand is experienced at a retailer and a shipment is received from the supplier. Transportation cost contains a fixed and a per unit charge. In
case the supplier does not have enough on-hand inventory to satisfy demand, an external supplier immediately brings the supplier’s inventory up to the required level.

Cachon and Fisher (1997) use one year of data obtained from Campbell Soup to create a simulation model of Campbell’s inventory policy. In the simulation, Campbell is assumed to have sufficient inventory to meet retailer demand and only ships full truckloads. The simulation is very detailed with four retailers, over 100 products, yearly seasonality, weekly seasonality, and product promotions that cause drastically increased sales. In the simulation, the supplier determines the quantity of each product to ship to each retailer every day. This decision is made using the retailers’ inventory levels and a forecast of demand during the lead time.

The simulation demonstrates that, had the policies outlined in this paper been instituted instead of a proposed retailer-managed inventory system, a 66% reduction in the retailers’ costs would have resulted. Although this is a large improvement to the retailer, the impact on Campbell Soup’s own inventory level is not considered. The authors conclude that the rules they developed are not needed by Campbell Soup since the same performance is achievable without centralized control if the retailers adopt their rules and place daily orders to Campbell Soup.

Waller, Johnson, and Davis (1999) simulate a supply chain having multiple distribution channels. The authors investigate the impact of reducing the ordering frequency from every four weeks to daily. The daily ordering is considered to be VMI. The authors consider the effect of several environmental factors and find that most of the benefit of VMI is the reduction of cycle stock due to more frequent ordering. It is also noted that demand variation has little effect on the benefits of VMI. Furthermore, the
authors find that VMI’s benefit is greatest when the number of participants in the VMI program is increased.

2.3 Empirical Research

The analytic results presented in the previous section indicate that information sharing and centralized control lead to important enhancements of supply chain performance. The empirical research demonstrates that information sharing and centralized control can indeed enhance supply chain performance, although there are conflicting results.

2.3.1 Case / Survey Research

Kulp, Lee, and Ofek (2004) investigate the impact of information sharing and manufacturer/retailer collaboration on the manufacturer’s profit margins. This was accomplished using a survey of senior executives in the Food & Packaged Consumer Goods industry. The respondents to the survey were intimately aware of the degree of collaboration and information sharing their companies had with its retailers.

The level of information sharing was based on the degree the two companies shared various types of information such as research information, information on new product demand, and information on the demand for new services. Respondents were also asked whether their retailers shared point-of-sale or inventory information. With respect to the degree of collaboration, respondents were asked whether they participated in VMI or the use of reverse logistics. Additionally, the degree of collaboration on new product development was measured. To measure profit margin, respondents compared
their firm’s margins to competing firms in the same industry. Finally, the survey inquired about the firm’s wholesale price and stockout rates, as well as their retailers’ stockout rates.

Each firm was classified as either a low, medium, or high performer based on its profit margins. Results show that the degree of information sharing distinguishes between average and below-average performers at a statistically significant level. Furthermore, VMI significantly separates above-average performer from average performers. The authors conclude from this that information sharing can improve system performance only up to a point, after which a collaborative effort, such as VMI, is needed to allow further improvements in performance.

Regression analysis was used to further analyze the results. Here it was found that both information sharing and VMI have a direct effect on profit margin and that this relationship was not mediated by stockout rates or price. Because the relationship is not mediated, an alternative explanation is needed to account for these relationships. The authors propose that improved production efficiencies might explain the result, although this was not tested in the paper.

Vergin and Barr (1999) performed a study of ten consumer products manufacturers who were currently using a continuous replenishment program with their customers. These companies reported that their customers experienced great results from VMI (inventories lowered by 32%, stockouts reduced by 55%). However, only two of the manufacturers improved production costs and only one improved its inventory levels. Interviews with company personnel indicated that many of the manufacturers felt the benefit their customers obtained were the result of the manufacturer carrying more
inventory themselves. Despite some negative results, eight of the ten manufacturers stated that continuous replenishment led to increased sales.

Clark and Hammond (1997) investigate whether supply chains adopting both process and technological improvements have better performance than channels adopting technological improvements alone. Here, technological improvements are representative of an EDI implementation, whereas process improvements encompass implementations of activities such as VMI or CRP. The performance of the supply chain is measured as the inventory turns needed to obtain a constant stockout level. The authors find that process improvements coupled with an EDI implementation results in improvements in inventory turns. However, firms adopting only technological improvements experienced no reductions in inventory levels.

### 2.3.2 Experimental Research

Steckel, Gupta, Banerji (2004) test whether shorter cycle times and shared point-of-sale information improve supply chain performance. The experiment consisted of day and evening MBA students playing the Beer Game, which in this case was limited to a three-level supply chain having a retailer, wholesaler, and distributor. Three factors were included in the experiment. The first factor was the delay between order placement and order delivery. This delay was varied at two levels; the order and shipping delays took either one period or two periods. The second factor explored the sharing of point-of-sale information. The two levels of this factor consisted of whether point-of-sale information was available to all supply chain participants or only the retailer. The authors were also interested in exploring how the pattern of customer demand affects the
benefit of decreased cycle time and the sharing of point-of-sale information. Thus, they included in their experimental design three separate demand functions that varied in shape and variability. The three demand functions investigated were the step-up function, an S-shaped demand with no error, and an S-shaped demand with error.

Supply chain performance was calculated as the sum of stockout and holding costs at each stage. Concerning the benefit of reducing the delay between order and delivery, the results show that there is a statistically significant difference in cost between the one-period and two-period delays. This is true for every level of the supply chain in both the step-up demand pattern and the S-shaped with no error demand pattern. With respect to the S-shaped with error demand pattern, there was a significant difference only at the retailer and wholesaler, not at the distributor. The authors conclude from this that improving cycle times makes a significant performance improvement, although the improvement may be moderated by the shape and variability of the demand distribution.

For the step-up and S-shaped with error demand functions, the authors found that sharing point-of-sale information improves the retailer’s cost at a statistically significant level. However, sharing point-of-sale information does not improve the wholesaler or distributor cost. For the S-shaped with no error demand function, the results indicate that information sharing increases cost for the distributor, but has no statistically significant affect on either the retailer or the wholesaler.
2.4 Stochastic Economic Lot Scheduling Problem

The Stochastic Economic Lot Scheduling Problem (SLSP) is concerned with determining control policies for the production of several products on a single machine in the situation where products may not be co-produced. Papers investigating situations with no setup time or cost tend to use base-stock policies and dynamic sequencing of production. Because an optimal policy is extremely difficult to calculate, most papers present heuristic policies. For a review of the literature in SLSP, refer to Sox et al. (1999) and for a review of ELSP, refer to Elmaghraby (1978).

Vergin and Lee (1978) investigate various scheduling rules that can be used to schedule multiple products on a single machine where products may not be co-produced and demand is stochastic. The first two rules determine cycle schedules assuming demand is deterministic. The third utilizes a ratio which compares the run length of an individual product relative to that of the other products. After manufacturing commences for a product, it continues until the ratio of that product’s inventory to the total inventory equals the previously calculated ratio. Production also terminates if inventory of another product reaches zero.

The remaining rules are based off the third. The fourth rule adds an additional constraint, where production is halted when a product’s inventory reaches a pre-specified maximum level. The fifth rule is an extension of both the fourth rule. In this case, when the decision of whether to change production between customers is made, the manufacturer first chooses a customer having backorders. If there are no customers with backorders, then the manufacturer will choose a customer having zero inventory. If all customers have positive inventory levels, production is halted as in the fourth rule. The
sixth rule builds on the fifth. As in the fifth rule, production is prioritized first for customers having backorders, then for customers having zero inventory. However, the sixth rule also switches production when a customer’s inventory level, measured in days of supply, falls below a minimum level.

The authors perform a numerical analysis of a periodic review environment having ten heterogeneous products and 85% utilization. Simulation is used to compare the performance of the previously mentioned rules. It is found that rules based on deterministic demand offer poor results. It is also noted that, between the third through sixth rules, the best policy depends on such factors as the heterogeneity and number of products, utilization, and the carrying, setup, and backorder costs.

Graves (1980) introduces a heuristic to solve the multiproduct production scheduling problem. Inventory is inspected periodically and production continues for an entire period. After production is complete, the inventory is not able to satisfy demand until after a fixed lead time. There are no setup times and excess demand is backordered. Costs include setup costs as well as holding and backorder costs.

Graves’s heuristic involves creating a “composite product” which is based on the aggregated inventory. The objective of the heuristic is to control congestion at the manufacturer. It is tested against several other heuristics, including some meant to represent (Q, R) and (s, S) policies. Results indicate that the composite product heuristic provides lower costs than all but one other policy, in which the results were inconclusive.

Federgruen and Katalan (1996) introduce a production strategy for the SLSP which they refer to as the periodic base-stock policy. In this policy, items are produced in a fixed sequence. Once manufacturing begins for a product, it continues until a batch
is completed or until a base-stock level is reached. After production for an item is finished, an idle time of a given length is inserted before setting up the next item. The idle time is inserted to reduce the frequency of setups and thereby reduce the average setup costs. The paper considers setup times and setup, backorder, and holding costs. Results of the periodic base-stock policy are compared to deterministic ELSP problems. The primary conclusion from this is that uncertainty in the production process causes dramatic increases in cost, especially at high levels of service.

Zheng and Zipkin (1990) consider the case where two products have identical distributions and follow a Poisson arrival pattern. The manufacturing time is drawn from an exponential distribution and this distribution is identical for each product. It is assumed all stockouts are backordered. The objective of the study is to determine which product to manufacture and at what time. Two models are considered. In the first, each product is controlled using a continuous review (S-1, S) model and orders are manufactured on a First-Come-First-Serve (FCFS) basis. In the second model, inventory is controlled with a continuous review (S-1, S) policy, but now the manufacturer is able to choose which product to manufacture based on their respective inventory levels. Using a numeric analysis, the authors demonstrate that the second model offers lower inventory levels and fewer backorders than the first.

Zipkin (1995) considers a situation similar to Zheng and Zipkin (1990). As before, there are no economies of scale in production and no setup times when switching between products. The products to be manufactured all have the same demand distribution. A base-stock policy is used such that production continues for a product until its inventory position reaches the base-stock level. Two models are used to decide
which product to manufacture. The first model follows a simple FCFS rule while the second manufacturers the product having the lowest inventory position. The performance of the rules are compared in a numeric analysis where the number of products (2 – 16), the utilization (25% - 90%), and the variance in production times (0.25 – 2.0) are varied.

Results indicate that there is little benefit to the second model over that of FCFS when utilization is low. However, the benefit increases and continues to grow even at utilizations up to 90%. The results also show that improvements are greatest when there are a large numbers of products. Finally, the variance in production times has only weak effect on the benefit of the second model, with the reduction in variance in the second model ranging from 14% to 19%.

Duenyas and Van Oyen (1996) consider setup times, setup costs, as well as holding and backorder costs while developing a policy for SLSP. They take a slightly unique perspective by viewing the jobs as arriving to a single server, with queues forming when the server is busy. The policy they develop acts in a greedy manner; when a product is being manufactured, production continues so long as the queue for this product is greater than zero. The policy also utilizes simple rules for idling and switching products.

The heuristic is compared to several other policies, including a gated policy (manufacture the number of products in queue at the time production starts) and an exhaustive policy (manufacture a product until its queue is empty). It is found that the heuristic is robust, performing better than the exhaustive or gated heuristic under all environmental conditions.
Perez and Zipkin (1997) consider a case with no setup time or setup costs when switching the single machine from one product to another. The authors utilize two policies to control inventory levels; an on-off policy and an allocation policy. The on-off policy determines the timing of production and the allocation policy specifies which product to produce. Because there are no setup costs, a base-stock on-off policy is used as the on-off policy.

Several policies are compared, including the static priority policy and a myopic policy that the authors develop. The myopic policy determines which product to manufacture based on an expected cost function over a time T. Using a numerical analysis in the setting where there are two identical products, it is found the myopic policy performs very well under most environments, it performs best in moderate traffic environments, and it provides solutions very close to optimal.

Johnson and Scudder (1999) consider a situation with small setup times to mimic the performance of a Quick Response system. Five scheduling rules are investigated, including (1) a lot-for-lot rule, (2) a fixed-cycle rule, (3) a lot-to-lot rule where priority is given based on days of supply, (4) a rule that optimizes the minimum number of days of inventory across all products, and (5) rule that maximizes the minimum short term availability of all products.

Experimental results demonstrate that policies 4 and 5 provide much better results than a lot-for-lot policy. In addition, rule 3, a modified lot-for-lot rule performs almost as well as the more complicated policies 4 and 5 in several environments. In addition, it is found that the greatest benefit of these rules over the simple lot-for-lot is found in environments having high utilization and high demand variability.
2.5 Comparison of Dissertation to Extant Literature

The literature investigates a number of mechanisms through which VMI and information sharing improve supply chain performance. Much of the literature addresses how VMI and information sharing enhance supply chain performance through improved replenishment timing (Gavirneni 2002, Gavirneni et al. 1999, Moinzadeh 2002). Other authors investigate how VMI can be used to reduce lead times and lot sizes (Cachon and Fisher 2000). Some investigate the impact of shipment consolidation, replenishment coordination, and stock rebalancing on supply chain performance (Cheung and Lee 2002). A few authors even examine how to structure VMI contracts to benefit supply chain participants (Fry et al. 2001, Cachon 2004).

Similar to much of the previous VMI research, in this dissertation we focus on how VMI may be used to improve production and replenishment timing. This dissertation, however, differs from the extant literature in several ways. While the majority of the literature compares VMI to a MTS replenishment strategy, it is not uncommon for VMI to be implemented in MTO environments. Consequently, the first study of this dissertation compares VMI to both MTS and MTO strategies in a serial supply chain. Furthermore, since we consider the vendor to be a capacitated manufacturer, we assume the manufacturer produces in batches and delays shipping an order if production for a batch is not complete.

Table 2.1 compares the first study of this dissertation to some of the most relevant literature along several dimensions. It appears from the table that our work is most similar to Chen (1998), however, Chen does not consider MTO policies and allows the
vendor to send partial shipments, unlike this study. Moinzadeh (2002) is also similar, however, an arborescent system is investigated while we consider a serial system.

Table 2.1 – Comparison of Study 1 to relevant literature

<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>Shipment Delays</th>
<th>Capacitated</th>
<th>Batch Ordering</th>
<th>Supply Chain Structure</th>
<th>Decision Epoch</th>
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<tr>
<td></td>
<td>MTS</td>
<td>MTO</td>
<td>Manufacturer</td>
<td>Serial</td>
<td>Arborescent</td>
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<td>Cachon &amp; Fisher (2000)</td>
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<td>Gavirneni (2002)</td>
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For the second study of this dissertation’s, the SLSP literature is most relevant. These papers examine the scheduling of multiple products on a single capacitated machine where the various products may not be co-produced. The SLSP literature often investigates cyclic and dynamic policies. Under a cyclic policy, the manufacturer follows a fixed production sequence so that the manufacturer always knows the order in which to manufacture the various products. Under a dynamic policy, however, once the manufacturer completes production for a certain product, it may choose to produce any product without constraint. In our second study, the heuristic we propose has the manufacturer choose products based on their inventory levels and hence is a dynamic policy.

Our work differs in several aspects to the majority of the SLSP literature. Most noticeably, we examine a situation where the manufacturer produces in fixed lot sizes that are not necessarily identical across products. Furthermore, we consider a situation
where there are no partial shipments. Consequently, the manufacturer must complete a production lot before shipping it to the customer.

Table 2.2 – Comparison of Study 2 to SLSP literature

<table>
<thead>
<tr>
<th>Fixed Lots</th>
<th>Nonidentical Lot Sizes</th>
<th>Complete Shipments</th>
<th>Leadtime</th>
<th>Policy Type</th>
<th>Decision Epoch</th>
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<td>Continuous</td>
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<td>Gallego (1990)</td>
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<td>Graves (1980)</td>
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<td>Perez &amp; Zipkin (1997)</td>
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<td>Zipkin (1995)</td>
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</table>

The second study is most similar to the work of Graves (1980) and Zipkin (1986). The major difference between our second study and the work of Graves is that Graves considers only situations where products have identical lot sizes while we allow for non-identical lot sizes. Furthermore, Graves allows the manufacturer to send partial shipments to the customers. In our second study we assume there is a lead time when shipping product from the manufacturer. This differs from Zipkin (1986) in that Zipkin does not consider shipping lead times, although the product is made available for use only after a batch has been completed. As well, Zipkin considers a situation where the manufacturer makes decisions continuously whereas we assume the manufacturer makes decisions periodically.
3. VENDOR MANAGED INVENTORY IN A SERIAL SUPPLY CHAIN

This chapter investigates how the critical component of a VMI program, giving the manufacturer the ability to specify the timing of production and shipments, offers improved performance in a two-echelon serial supply chain. We compare three replenishment strategies (1) MTO, (2) MTS, and (3) VMI. The comparison with an MTO replenishment strategy is significant because there are few previous studies that compare its performance to such supply chain initiatives as VMI (Sahin and Robinson 2005). For MTO and MTS strategies, we assume the customer places orders based on a continuous review (s, S) policy. Under VMI, the manufacturer manipulates the production signal, $s_1$, and shipping signal, $s_2$ to achieve the desired system performance. We perform analysis numerically and through simulation to augment our theoretic result.

3.1 Introduction

In a traditional supply chain, a manufacturer receives customer inventory information only when the customer (e.g. a retailer) places an order. Because the manufacturer is unsure when an order will be placed, the choice of production strategies is limited. Depending on the customer's requirements, we first consider that the manufacturer may follow either a Make-to-Stock (MTS) strategy, or a Make-to-Order (MTO) strategy. Under MTS, the manufacturer produces in anticipation of demand and carries finished goods inventory from the time production is completed until an order is placed. Under MTO, the manufacturer ships an order immediately after its production is completed. Consequently, the manufacturer holds less inventory when employing an MTO rather than an MTS strategy. Although, MTS requires the manufacturer to hold
more inventory, the customer needs to carry less inventory because of the shorter lead time.

Rather than being restricted to choosing between MTO and MTS strategies and accepting the manufacturer and customer inventory levels that result, a VMI strategy might offer the manufacturer more flexibility. Under VMI, the manufacturer has access to customer inventory information and determines the timing of both production and shipments. This enables the manufacturer to manipulate the placement of inventory across the supply chain in ways not possible under MTS or MTO. Note that VMI can also be implemented in a manner equivalent to either an MTO or MTS strategy.

This chapter investigates how the critical component of a VMI program, giving the manufacturer the ability to specify the timing of production and shipments, offers improved performance in a two-echelon serial supply chain. Three models are compared which represent the three replenishment strategies (1) MTO, (2) MTS, and (3) VMI. The three models represent different options of where inventory is placed in the supply chain. Compared to MTS, where the manufacturer carries finished goods inventory, under MTO the manufacturer holds less inventory because shipments are sent to the customer immediately after production terminates. This, however, necessitates the customer to carry more inventory due to MTO’s longer lead time. VMI, on the other hand, offers intermediate levels of inventory at both the customer and the manufacturer. It is significant that MTO is included in this study because typical supply chain literature looks only at the benefit of information sharing and centralized control relative to MTS systems (Sahin and Robinson 2005).
Section 3.2 describes the supply chain configuration and details how inventory is controlled under each replenishment strategy. In Section 3.3 the analytic models used to calculate exact results for expected inventory levels and fill rates for a specific level of \( s \) and/or \( s_1 \) and \( s_2 \) are presented. The results of a numeric analysis of the factors are presented in Section 3.4. Because the analysis becomes onerous at high utilizations, Section 3.5 discusses and presents the results of a simulation where the effect of several environmental factors are explored. Finally, the significance of the results is discussed in Section 3.6.

3.2 Supply Chain, Replenishment Models, and Performance Measures Description

This study considers a two-stage serial supply chain where a single manufacturer replenishes a single customer. Here, the customer orders from a manufacturer using a continuous review \((s, S)\) inventory policy. Because continuous review is assumed, the difference in the two parameters, \( S-s \), will always yield a constant order size, \( Q \). The order size is exogenous, as in Chen (1998) and De Bodt and Graves (1985), so that only the reorder point is a decision variable. The manufacturer is modeled as having a fixed production rate, \( p \), and ships only complete orders. The manufacturer incurs no setup cost and produces batches of size \( Q \); in other words the manufacturer follows a lot-for-lot strategy. Once the manufacturer sends a shipment to the customer, the transportation lead time, \( L \), is constant. When demand exceeds available inventory, the excess demand is backordered and is satisfied at the next replenishment.

Model 1 represents an MTO replenishment strategy whereby the customer continuously monitors its inventory levels and places an order for \( Q \) units when its
inventory position reaches \( s \). After receiving an order, the manufacturer immediately begins production of the order if capacity is available; otherwise the order is delayed until the manufacturer completes its current production run. Once production has begun, the order is complete in \( P = \frac{Q}{p} \) time units, at which point the manufacturer ships the entire order to the customer. Consequently, the manufacturer holds no finished goods inventory. Figure 3.1 displays the ordering pattern for Model 1.

*Figure 3.1 – Model 1 (MTO) ordering pattern and inventory levels*

Model 2 corresponds to an MTS strategy. Again, the customer reviews its inventory levels continuously, placing an order for \( Q \) units when its inventory position reaches \( s \). If the manufacturer has \( Q \) units of inventory available, a shipment is sent immediately and arrives at the customer’s site in \( L \) time units. If \( Q \) units are not available, this implies that a production run has yet to be completed. Consequently, the shipment is sent after the run is completed. In either case, after the manufacturer ships a
customer order, a new production run commences, bringing the manufacturer’s inventory level back to Q units. Thus, the manufacturer carries finished goods inventory. This is shown in Figure 3.2.

*Figure 3.2 – Model 2 (MTS) ordering pattern and inventory levels*

![Diagram showing Model 2 (MTS) ordering pattern and inventory levels.]

Under a VMI strategy, Model 3, the manufacturer continuously monitors the customer’s inventory level and determines the timing of production and shipment based on this level. When the customer’s inventory position reaches $s_1$, the manufacturer immediately begins producing Q units. The manufacturer holds the Q units in finished goods inventory until the customer’s inventory position falls to $s_2$, at which point the manufacturer ships all Q units. As in Model 2, if the customer’s inventory reaches $s_2$ and the manufacturer does not have Q units, the shipment to the customer is delayed until a complete order is available.
For all three models, the performance measures are the average customer inventory, the average manufacturer inventory, and the average fill rate. To directly compare the results across the three models, the average inventory levels will be determined for one of several fill rate levels. The actual $s$, $s_1$, and $s_2$ parameters necessary to reach these fill rate targets will be determined using a quadratic search algorithm. As discussed later in this chapter, VMI may have multiple solutions for one fill rate.
3.3 Analytic Development

This research analytically calculates inventory and service levels which results in each of the three replenishment models. More specifically, for MTO and MTS, the analytic models calculate the manufacturer and customer inventory levels as well as the expected fill rate that result for a given value of $s$ (the customer reorder point) and $Q$ (the order quantity). For VMI, the models calculate the supply chain performance given values of $s_1$ (production trigger), $s_2$ (shipping trigger), and $Q$.

The calculations of the analytic models are based on renewal theory. A process renews when it reaches a pre-specified state, called the renewal point. An important characteristic of a renewal point is that the probabilities of future outcomes are the same each time this point is reached. With respect to the inventory system studied here, a state is defined as a specific combination of customer and manufacturer inventory levels. Renewal points for each model were chosen based on their analysis tractability. For an MTO environment, a renewal occurs when the customer’s inventory level, $I_c$, reaches $s$ and the manufacturer’s inventory level, $I_m$, is zero ($I_c = s$, $I_m = 0$). For an MTS environment, a renewal occurs when the customer’s inventory reaches $s$ and the manufacturer has a completed lot available ($I_c = s$, $I_m = Q$). Under a VMI model, a renewal occurs when the manufacturer’s inventory level is equal to $s_1$ and the manufacturer has no inventory available ($I_c = s_1$, $I_m = 0$).

Because there is a fixed production time during which the manufacturer may not ship, the system will not necessarily renew with each replenishment. The system does not renew when, having started at a renewal point, the demand during the production run exceeds $Q$. Because the process does not necessarily renew at each replenishment, there
are a random number of replenishments sent from the manufacturer to the customer between system renewals. To calculate the various performance measures, it is first necessary to determine the expected number of replenishments that occur during the time between successive renewals.

### 3.3.1 Expected Number of Replenishments between Renewals

Under continuous review, the interarrival time between demands is assumed to be exponential with rate $\lambda$. When calculating the performance measures, however, it is necessary to know the demand distribution over certain time periods, such as the production time and the shipping lead time. Because interarrival times are exponential, the demand over a period of time is Poisson distributed with probability density function.

$$p(x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

Where

- $\lambda$ = the mean demand over the relevant timeframe
- $x$ = a random variable representing the demand during the relevant timeframe

Recall that a renewal does not occur if demand during a production run exceeds $Q$. Recognizing that, if a renewal has not taken place after the first production run, the manufacturer immediately begins producing another order. This results in the manufacturer performing consecutive production runs. The manufacturer continues to make consecutive production runs until the customer’s inventory at the end of a production run is greater than or equal to $s$. Let $Y$ represent the number of consecutive production runs since the system was last at its renewal point. In this case, the
probability of the system having precisely $Y = y$ consecutive production runs before a renewal occurs is as follows:

$$P(Y = 1) = \sum_{x_1 = 0}^{Q} p(x_1)$$

$$P(Y = 2) = \sum_{x_1 = Q}^{2Q} \sum_{x_2 = 0}^{2Q-x_1} p(x_1) p(x_2)$$

$$P(Y = 3) = \sum_{x_1 = Q}^{3Q} \sum_{x_2 = 2Q-x_1}^{3Q-x_1} \sum_{x_3 = 0}^{3Q-x_1-x_2} p(x_1) p(x_2) p(x_3)$$

Where

$x_Y =$ demand over production time for production run $Y$, $Y = 1, 2, 3, \ldots$

$Q =$ order quantity which is equal to $S - s$

To explain the logic for the above calculations, consider the equation for $Y = 1$ in an MTO system. Starting at the renewal point, $I_c = s$ and $I_m = 0$, if demand were greater than $Q$ during the production run, the customer’s inventory position would have reached $s$ while the manufacturer was producing. Hence, at $I_c = s$, $I_m < Q$ and no renewal occurs. Now consider the calculations for $Y = 2$. Recognize that for it to take precisely two order cycles for a renewal to occur, it is necessary that a renewal does not occur during the first cycle (the first summation). For the second order to renew, observe that at the beginning of the second production run, $I_c$ is already $Q-x_1$ units below $s$. Thus, for a renewal to occur, demand must be less than $2Q-x_1$ units (the second summation). For $Y>2$, the same pattern of calculations hold.
After determining the probability \((Y = y)\), the expected number of production runs, and therefore replenishments, between renewals may be calculated as follows.

\[
\sum_{y=1}^{\infty} y \cdot p(y)
\]

### 3.3.2 MTO and MTS Models

**Expected Units Short Per Renewal**

One performance measure is the expected number of backorders the customer experiences between renewals, which we refer to as the Expected Units Short Per Renewal (EUSPR). The calculation for MTS and MTO are identical, except that MTO has a longer lead time due to the production time. Thus, \(t = P + L\) when an MTO policy is used and \(t = L\) under MTS.

Since there may be several replenishments between each renewal, the number of units short must be calculated for each. Let \(\text{EUSPR}_y\) represent the contribution of the shortage for the \(Y^{th}\) order cycle to the overall EUSPR calculation. For \(Y = 1\), \(\text{EUSPR}\) calculates the average shortage over \(t\) given that the shipment is sent (MTS) or production is initiated (MTO) when the customer inventory is at \(s\). For \(Y = 2\), the demand during the first production run, \(x_1\), must be greater than \(Q\) for the process to not renew. The first summation considers the values of \(x_1\) that meet this condition and their probability of occurring. Given an initial inventory position of \((s + Q - x_1)\), the second summation calculates the expected shortage over lead time \(t\). In fact, for any value of \(Y = y\), the last summation always calculate the shortage over time \(t\), given that the customer inventory level is \((s + (y-1)Q - x_1 - x_2 - \ldots - x_{y-1})\).
Starting at the renewal point, the expected shortage for the first three replenishments is calculated below. Note that the above pattern holds for Y > 3, i.e., each additional order will require another summation and an additional variable.

\[ \text{EUSPR}_1 = \sum_{x_1 = s+1}^{\infty} (x_1 - s)p(x_1) \]

\[ \text{EUSPR}_2 = \sum_{x_1 = Q+1}^{\infty} \sum_{x_1 = s+Q-x_1+1}^{\infty} [x_1 - (s + Q - x_1)]p(x_1)p(x_2) \]

\[ \text{EUSPR}_3 = \sum_{x_1 = Q+1}^{\infty} \sum_{x_2 = 2Q-x_1+1}^{\infty} \sum_{x_1 = s+2Q-x_1-x_2+1}^{\infty} [x_1 - (s + 2Q - x_1 - x_2)]p(x_1)p(x_2)p(x_3) \]

Where

\[ x_t = \text{demand over time } t, \text{ where } t = L+P \text{ for MTO systems and } t = L \text{ for MTS systems} \]

\[ x_Y = \text{demand over production time for production run } Y, \ Y = 1, 2, 3, \ldots \]

By summing the expected shortages for each value of \( y \), the total expected shortage between each renewal is determined. However, it would be more useful to know the expected shortage per replenishment cycle (ESPRC), a more common measure. This is obtained by dividing EUSPR by the expected number of orders between renewals.

\[ \text{ESPRC} = \frac{\sum_{y=1}^{\infty} \text{EUSPR}_y}{\sum_{y=1}^{\infty} y \cdot p(y)} \]

Once ESPRC is calculated, the fill rate of the system may be calculated as follows:

\[ \text{Fill Rate} = 1 - \frac{\text{ESPRC}}{Q} \]
Customer’s Average Inventory

The customer’s inventory can be divided into safety stock and cycle stock. For any value of \( Y \), the average cycle stock is simply \((S-s)/2\). Safety stock, defined as the average amount of inventory remaining at the customer immediately before the replenishment arrives, requires considering whether demand during the \( Y^{th} \) production cycle is greater than \( Q \). Note that we consider a case where safety stock can be negative. For \( Y = 1 \) to 3, the safety stock calculations are shown below. For \( Y > 3 \), the calculations follow the same general form.

\[
SS_1 = \sum_{x_l=0}^{\infty} (s - x_l)p(x_l)
\]

\[
SS_2 = \sum_{x_1=Q+1}^{\infty} \sum_{x_l=0}^{\infty} (s + Q - x_l - x_1)p(x_1)p(x_l)
\]

\[
SS_3 = \sum_{x_1=Q+1}^{\infty} \sum_{x_2=2Q-x_1}^{\infty} \sum_{x_l=0}^{\infty} (s + 2Q - x_1 - x_2 - x_l)p(x_1)p(x_2)p(x_l)
\]

Similar to the ESPRC calculation, the total expected safety stock, \( SS \), is calculated as the expected safety stock per renewal divided by the expected number of replenishments per renewal.

\[
SS = \frac{\sum_{y=1}^{\infty} SS_y}{\sum_{y=1}^{\infty} y \cdot p(y)}
\]

If the safety stock is positive, then an approximation for the average customer inventory is simply half the lot size plus the safety stock, as shown below.
When \( SS \geq 0 \)

\[
I_c = SS + \frac{Q}{2}
\]

If safety stock is negative, however, the customer inventory calculation must consider the proportion of time over the replenishment cycle that inventory is zero. This is accomplished by first noting that, because safety stock is the average amount of inventory on-hand immediately before a replenishment arrives, then the stock on-hand will be \((SS + Q)\) after a replenishment arrives. Because the interarrival time of demand is \(1/\lambda\), then the amount of time it then takes for this quantity of inventory to equal zero is \((SS + Q)/\lambda\). During this time, the average inventory level is \((SS + Q)/2\). Because a cycle lasts an average time of \(Q/\lambda\), an approximation for the long run average customer inventory is as follows.

When \( SS < 0 \)

\[
I_c = \frac{(Q + SS)^2}{2Q}
\]

**Manufacturer Average Inventory**

To calculate the manufacturer’s average inventory, first consider that the manufacturer is in one of two states; it is either producing or idle. During its production time, the average inventory is equal to \(Q/2\) units. After production, the manufacturer immediately ships the completed order in an MTO system but holds the inventory under an MTS system. Thus, during the manufacturer’s idle time no inventory is held under MTO whereas \(Q\) units are held under MTS. Recognizing that the utilization of the
system, \( \lambda/p \), represents the proportion of time the manufacturer is producing, we can calculate \( I_m \) for an MTO system as:

\[
I_m = \frac{\lambda Q}{2p} + 0 \left( 1 - \frac{\lambda}{p} \right) = \frac{\lambda Q}{2p}
\]

And for an MTS system it is:

\[
I_m = \frac{\lambda Q}{2p} + Q \left( 1 - \frac{\lambda}{p} \right) = \frac{Q(2p - \lambda)}{2p}
\]

### 3.3.3 Vendor Managed Inventory Systems

Recall that under a VMI strategy, Model 3, the manufacturer begins producing \( Q \) units when the customer’s inventory position reaches \( s_1 \). The manufacturer then holds the \( Q \) units in finished goods inventory until the customer’s inventory position falls to \( s_2 \), at which point the manufacturer ships all \( Q \) units. The manufacturer does not begin another production cycle until \( s_1 \) is reached again. As in Model 2, if the customer’s inventory reaches \( s_2 \) and the manufacturer does not have \( Q \) units, the shipment to the customer is delayed until a complete order is available.

**Expected Units Short Per Renewal**

So long as the demand during the first production run is less than \( Q = (s_1 - s_2) \) the manufacturer will have \( Q \) units on-hand when the customer’s inventory reaches \( s_2 \). Define this as \( \text{EUSPR}_{1a} \). If demand is greater than this, the manufacturer will have less than \( Q \) units and it is necessary to determine the expected number of units on-hand after the production run is ended. Define this as \( \text{EUSPR}_{1b} \). For the first production run the calculations for ESPRC are:
For \( Y = 2 \), the calculation for EUSPR requires three separate calculations. The calculations for EUSPR_{2a} and EUSPR_{2b} represent situations where demand during the first production run is greater than \( Q \), but not so large that it is not possible for the customer’s inventory to be greater than \( s_2 \) during the second production run. The situation where demand during the second production run is low enough that the customer’s inventory has yet to reach \( s_2 \) at the end of the run is calculated in EUSPR_{2a} whereas EUSPR_{2b} considers the other scenario. EUSPR_{2c} considers the case where demand is so large during the first production run that it is impossible for the system to be above \( s_2 \) at the end of the second run (and thus eliminating the possibility that probability distribution is calculated with a lead time of \( L \)).

\[
\text{EUSPR}_{1a} = \sum_{x_1 = 0}^{s_1-s_2} p(x_1) \sum_{x_L = s_2+1}^{\infty} (x_L - s_2) p(x_L)
\]

\[
\text{EUSPR}_{1b} = \sum_{x_1 = s_1-s_2}^{\infty} \sum_{x_L = s_1-x_1+1}^{\infty} \left[ x_L - (s_1 - x_1) \right] p(x_1) p(x_L)
\]

\[
\text{EUSPR}_{2a} = \sum_{x_1 = Q+1}^{s_1-s_2+Q} \sum_{x_2 = 0}^{s_1-s_2-x_1+Q} p(x_1) p(x_2) \sum_{x_L = s_2+1}^{\infty} (x_L - s_2) p(x_L)
\]

\[
\text{EUSPR}_{2b} = \sum_{x_1 = Q+1}^{s_1-s_2+Q} \sum_{x_2 = s_1-s_2-x_2+Q+1}^{s_1-x_1+Q-x_2} \sum_{x_L = s_1-x_1+Q-x_2}^{\infty} \left[ x_L - (s_1 - x_1 + Q - x_2) \right] p(x_1) p(x_2) p(x_L)
\]

\[
\text{EUSPR}_{2c} = \sum_{x_1 = s_1-s_2+Q}^{\infty} \sum_{x_2 = s_1+Q-x_1}^{\infty} \left[ x_{LP} - (s_1 + Q - x_1) \right] p(x_1) p(x_{LP})
\]

\[x_L = x_{LP} = \]
The same logic applies to values of \( Y \) greater than 2; multiple scenarios must be calculated that consider the chance that the number of units short during the \( Y^{th} \) replenishment equals (1) \( x_L - s_2 \), (2) \( x_L - (s_1 - x_1 - x_2 - \ldots - x_Y + (Y - 1) \times Q) \), or (3) \( x_{LP} - (s_1 + (Y - 1) \times Q - x_1 - x_2 - \ldots - x_{Y-1}) \). The final ESPRC is calculated in the same manner as those for MTS and MTO.

**Average Customer Inventory**

The average customer inventory for VMI are shown below for \( Y = 1 \) and \( 2 \). Note again it is necessary for demand to be greater than \( Q \) to cause consecutive production runs. The explanation of the safety stock calculation for \( Y > 1 \) is similar to the EUSPR calculation for the same \( Y \). In other words, it considers three scenarios: (1) at the end of the second production run \( I_m = Q \) and \( I_c \geq s_2 \), (2) at the end of the second production run \( I_m = Q \) and \( I_c < s_2 \), (3) demand during the first production makes it impossible for the customer inventory to exceed \( s_2 \) at the end of the second production run.

\[
SS_{1a} = \sum_{x_1 = 0}^{s_1 - s_2} p(x_1) \sum_{x_L = 0}^{s_2} (s_2 - x_L) p(x_L)
\]

\[
SS_{1b} = \sum_{x_1 = s_1 - s_2}^{\infty} \sum_{x_L = 0}^{s_1 - x_1} (s_1 - x_1 - x_L)p(x_1)p(x_L)
\]
\[
SS_{2a} = \sum_{x_1 = Q+1}^{\infty} \sum_{x_2 = 0}^{s_1-s_2+Q} \sum_{x_L = 0}^{\infty} (s_2 - x_L)p(x_L)
\]

\[
SS_{2b} = \sum_{x_1 = Q+1}^{\infty} \sum_{x_2 = s_1-s_2+Q+1}^{\infty} \sum_{x_L = 0}^{\infty} [(s_1 - x_1 + Q - x_2) - x_L]p(x_1)p(x_2)p(x_L)
\]

\[
SS_{2c} = \sum_{x_1 = s_1-s_2+Q}^{\infty} \sum_{x_L = 0}^{\infty} [(s_1 + Q - x_1 - x_{LP})]p(x_1)p(x_{LP})
\]

To calculate the total expected safety stock per replenishment cycle and the average customer inventory, the following calculations are needed.

\[
SS = \sum_{y = 1}^{\infty} SS_y \div \sum_{y = 1}^{\infty} y \cdot p(y)
\]

When \( SS \geq 0 \)

\[
I_c = SS + \frac{Q}{2}
\]

When \( SS < 0 \)

\[
I_c = \frac{(Q + SS)^2}{2Q}
\]
Average Manufacturer Inventory

Calculating the average manufacture inventory, $I_m$, is more involved than in the MTO and MTS models. Whereas those models considered only two time periods, production and non-production times, the VMI model must consider three distinct periods. Besides holding $Q/2$ units of inventory over the production time, under VMI the manufacturer holds $Q$ units from the time production is complete until the customer’s inventory level reaches $s_2$ and holds no inventory from the time a shipment is made until the customer’s inventory position reaches $s_1$.

We solve the problem by recognizing that an average cycle lasts $Q/\lambda$ time units. Of this time, the average inventory is $Q/2$ for the duration of the production run. Because the expected time for the inventory to decrease from $s_1$ to $s_2$ is less than the cycle time, the manufacturer holds $Q$ units of inventory for an average of $[(s_1 - s_2)/\lambda - Q/p]$ time units. The rest of the cycle time has zero units of inventory and so does not need to be included in the calculation.

When $C > (s_1-s_2)/\lambda$

$$\frac{Q}{2}P + \left(\frac{s_1 - s_2}{\lambda} - P\right)Q$$

Where

$P = \text{production time} = Q/p$

$C = \text{cycle time} = Q/\lambda$
3.4 Numerical Analysis

3.4.1 Factor Levels

To determine the effect that several environmental variables have on the performance of VMI, MTO, and MTS systems, a numerical analysis was performed using the calculations outlined in the previous section. The environmental factors and their levels are displayed in Table 3.1.

Table 3.1 – Numerical Analysis Factor Levels

<table>
<thead>
<tr>
<th>Factor</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot Size</td>
<td>10, 20, 30, 40</td>
</tr>
<tr>
<td>Utilization</td>
<td>50%, 60%, 70%, 80%</td>
</tr>
<tr>
<td>Lead Time</td>
<td>1, 2, 3, 4</td>
</tr>
<tr>
<td>Fill Rate</td>
<td>70%, 80%, 90%, 98%</td>
</tr>
</tbody>
</table>

All calculations have an arrival rate, \( \lambda \), equal to 10. This arrival rate was chosen because of the time required to perform calculations increases exponentially with increasing lambda. In addition, \( \lambda \) is kept small so that the coefficient of demand variation is kept relatively high. Because the number of arrivals over a time period is Poisson distributed, the coefficient of variation is \( \lambda^{-0.5} \), which implies that the coefficient of variation decreases as \( \lambda \) increases.

To calculate the desired performance measures efficiently, we developed a program coded using Microsoft Visual Basic for Applications to iteratively calculate the value of the performance measures for all values of \( Y \leq 4 \). In addition to the performing the calculations, the code also includes a quadratic search algorithm that finds the reorder point, production signal, and shipping signal that deliver the desired fill rate.

Because VMI has two parameters, several combinations of \( s_1 \) and \( s_2 \) will produce the desired fill rate. Rather than having the quadratic search algorithm choose one of these combinations arbitrarily, the value of \( s_2 \) is predetermined and then the quantity for
s\textsubscript{1} which is needed to obtain the target fill rate is found. The quantity used for the s\textsubscript{2} parameter is based on the results for a comparable MTO system having the same fill rate and factors levels. Specifically, after finding the value of \( s \) required for the comparable MTO system, \( s\textsubscript{2} \) is set as the average amount of inventory the customer has on hand when the MTO manufacturer completes production and ships the order to the customer.

The logic behind this method of determining the \( s\textsubscript{1} \) and \( s\textsubscript{2} \) values is that, if demand during the VMI production run is high, then the VMI manufacturer will ship at the same time as an MTO manufacturer, thereby minimizing the inventory held while providing an equivalent service level. On the other hand, if demand during the VMI production cycle is low, rather than shipping an order to a customer who is in no risk of stocking out, the manufacturer instead retains the inventory and waits for the customer’s inventory level to fall to the point that a replenishment is needed. Intuitively, compared to an MTO system, the customer’s average inventory should be reduced while not drastically increasing the inventory held at the manufacturer. Consequently, this policy is reasonable for situations where the customer’s inventory has higher holding costs than the manufacturer’s inventory.

### 3.4.2 Numeric Results

Tables 3.2 to 3.4 display the analytic results for MTO, VMI, and MTS systems. A simulation of a two-echelon supply chain having the same Poisson arrival process and the same replenishment strategies, i.e. VMI, MTO, and MTS, was also developed. For each policy and factor level combination, 5,000 order replenishments were simulated to obtain results for safety stock, ESPRC, customer and manufacturer inventories, and fill
rate. Common random numbers were used for each model. The simulation results are also included in Tables 3.2 to 3.4. The fact that the analytic and simulated results are comparable lends credibility to the simulation.

### Table 3.2 – Safety Stock Comparison of MTO, MTS, and VMI

<table>
<thead>
<tr>
<th>Factor</th>
<th>Levels</th>
<th>MTO</th>
<th>MTS</th>
<th>VMI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Analytic Results</td>
<td>Simulated Results</td>
<td>Analytic Results</td>
</tr>
<tr>
<td>Lot Size</td>
<td>10</td>
<td>1.90 (0.07)</td>
<td>2.44 (0.08)</td>
<td>1.81 (0.07)</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>-1.04 (-0.05)</td>
<td>-0.21 (0.09)</td>
<td>-0.95 (0.08)</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>-3.19 (-0.07)</td>
<td>-2.11 (0.10)</td>
<td>-3.05 (0.10)</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>-5.03 (-0.07)</td>
<td>-3.92 (0.11)</td>
<td>-4.91 (0.08)</td>
</tr>
<tr>
<td>Utilization</td>
<td>50%</td>
<td>-1.90 (-0.07)</td>
<td>-1.30 (0.09)</td>
<td>-1.75 (0.08)</td>
</tr>
<tr>
<td></td>
<td>60%</td>
<td>-1.89 (-0.07)</td>
<td>-1.14 (0.09)</td>
<td>-2.20 (0.07)</td>
</tr>
<tr>
<td></td>
<td>70%</td>
<td>-1.90 (-0.07)</td>
<td>-0.88 (0.10)</td>
<td>-1.97 (0.07)</td>
</tr>
<tr>
<td></td>
<td>80%</td>
<td>-1.68 (-0.08)</td>
<td>-0.48 (0.10)</td>
<td>-1.18 (0.08)</td>
</tr>
<tr>
<td>Lead Time</td>
<td>1</td>
<td>-2.87 (-0.05)</td>
<td>-1.90 (0.07)</td>
<td>-2.80 (0.05)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-2.09 (-0.07)</td>
<td>-1.24 (0.09)</td>
<td>-2.07 (0.07)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-1.52 (-0.08)</td>
<td>-0.61 (0.10)</td>
<td>-1.39 (0.08)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-0.88 (-0.09)</td>
<td>-0.05 (0.11)</td>
<td>-0.84 (0.09)</td>
</tr>
<tr>
<td>Fill Rate</td>
<td>70%</td>
<td>-7.26 (-0.07)</td>
<td>-7.09 (0.09)</td>
<td>-7.58 (0.07)</td>
</tr>
<tr>
<td></td>
<td>80%</td>
<td>-4.33 (-0.07)</td>
<td>-3.86 (0.09)</td>
<td>-4.59 (0.07)</td>
</tr>
<tr>
<td></td>
<td>90%</td>
<td>-0.68 (-0.07)</td>
<td>0.22 (0.09)</td>
<td>-0.74 (0.08)</td>
</tr>
<tr>
<td></td>
<td>98%</td>
<td>4.91 (0.07)</td>
<td>6.93 (0.09)</td>
<td>5.81 (0.08)</td>
</tr>
</tbody>
</table>

### Table 3.3 – ESPRC Comparison of MTO, MTS, and VMI

<table>
<thead>
<tr>
<th>Factor</th>
<th>Levels</th>
<th>MTO</th>
<th>MTS</th>
<th>VMI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Analytic Results</td>
<td>Simulated Results</td>
<td>Analytic Results</td>
</tr>
<tr>
<td>Lot Size</td>
<td>10</td>
<td>1.65 (0.04)</td>
<td>1.86 (0.04)</td>
<td>1.70 (0.04)</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>3.20 (0.05)</td>
<td>3.45 (0.06)</td>
<td>3.34 (0.05)</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>4.75 (0.06)</td>
<td>4.91 (0.07)</td>
<td>4.94 (0.06)</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>6.26 (0.06)</td>
<td>6.43 (0.08)</td>
<td>6.49 (0.06)</td>
</tr>
<tr>
<td>Utilization</td>
<td>50%</td>
<td>3.95 (0.05)</td>
<td>4.02 (0.06)</td>
<td>4.14 (0.05)</td>
</tr>
<tr>
<td></td>
<td>60%</td>
<td>3.94 (0.05)</td>
<td>4.07 (0.06)</td>
<td>4.34 (0.07)</td>
</tr>
<tr>
<td></td>
<td>70%</td>
<td>3.97 (0.05)</td>
<td>4.12 (0.07)</td>
<td>4.13 (0.05)</td>
</tr>
<tr>
<td></td>
<td>80%</td>
<td>3.99 (0.05)</td>
<td>4.45 (0.07)</td>
<td>3.85 (0.06)</td>
</tr>
<tr>
<td>Lead Time</td>
<td>1</td>
<td>3.92 (0.04)</td>
<td>4.14 (0.06)</td>
<td>4.15 (0.04)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.93 (0.05)</td>
<td>4.16 (0.06)</td>
<td>4.11 (0.05)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.02 (0.06)</td>
<td>4.16 (0.07)</td>
<td>4.07 (0.06)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3.99 (0.06)</td>
<td>4.18 (0.07)</td>
<td>4.12 (0.06)</td>
</tr>
<tr>
<td>Fill Rate</td>
<td>70%</td>
<td>7.87 (0.07)</td>
<td>8.05 (0.08)</td>
<td>7.93 (0.07)</td>
</tr>
<tr>
<td></td>
<td>80%</td>
<td>5.12 (0.06)</td>
<td>5.35 (0.08)</td>
<td>5.34 (0.06)</td>
</tr>
<tr>
<td></td>
<td>90%</td>
<td>2.55 (0.05)</td>
<td>2.70 (0.06)</td>
<td>2.65 (0.05)</td>
</tr>
<tr>
<td></td>
<td>98%</td>
<td>0.51 (0.02)</td>
<td>0.55 (0.03)</td>
<td>0.54 (0.03)</td>
</tr>
</tbody>
</table>
Table 3.4 – Avg. Customer Inventory comparison of MTO, MTS, and VMI

<table>
<thead>
<tr>
<th>Factor</th>
<th>Levels</th>
<th>MTS</th>
<th>MTO</th>
<th>VMI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Analytic</td>
<td>Simulated</td>
<td>(std. err.)</td>
</tr>
<tr>
<td>Lot Size</td>
<td>10</td>
<td>6.96</td>
<td>7.88 (0.06)</td>
<td>7.48</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>9.25</td>
<td>10.11 (0.05)</td>
<td>10.05</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>12.32</td>
<td>13.10 (0.05)</td>
<td>13.35</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>15.66</td>
<td>16.35 (0.05)</td>
<td>16.73</td>
</tr>
<tr>
<td>Utilization</td>
<td>50%</td>
<td>10.99</td>
<td>11.83 (0.05)</td>
<td>11.57</td>
</tr>
<tr>
<td></td>
<td>60%</td>
<td>11.00</td>
<td>11.83 (0.05)</td>
<td>11.73</td>
</tr>
<tr>
<td></td>
<td>70%</td>
<td>10.99</td>
<td>11.82 (0.05)</td>
<td>11.97</td>
</tr>
<tr>
<td></td>
<td>80%</td>
<td>11.21</td>
<td>11.96 (0.05)</td>
<td>12.35</td>
</tr>
<tr>
<td>Lead Time</td>
<td>1</td>
<td>10.05</td>
<td>10.66 (0.03)</td>
<td>10.99</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10.80</td>
<td>11.56 (0.04)</td>
<td>11.63</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>11.36</td>
<td>12.25 (0.06)</td>
<td>12.22</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>11.97</td>
<td>12.96 (0.07)</td>
<td>12.77</td>
</tr>
<tr>
<td>Fill Rate</td>
<td>70%</td>
<td>6.31</td>
<td>7.27 (0.04)</td>
<td>6.44</td>
</tr>
<tr>
<td></td>
<td>80%</td>
<td>8.59</td>
<td>9.51 (0.05)</td>
<td>9.00</td>
</tr>
<tr>
<td></td>
<td>90%</td>
<td>11.88</td>
<td>12.67 (0.05)</td>
<td>12.74</td>
</tr>
<tr>
<td></td>
<td>98%</td>
<td>17.41</td>
<td>17.98 (0.06)</td>
<td>19.43</td>
</tr>
</tbody>
</table>

The results indicate that MTO offers the lowest manufacturer inventory for all factor levels while MTS offers the highest. The converse is also true; MTS provides the lowest customer inventory while MTO yields the highest. VMI provides intermediate levels of both, but it provides value in the sense that it appears to offer much lower levels of manufacturer inventory while carrying only slightly more customer inventory than the
MTS system. Compare, for example, the difference in inventory levels between MTS and VMI for the lot size factor. VMI offers only slightly more customer inventory (approximately 0.1 units less for all factor levels), but it offers substantially less manufacturer inventory (7 units less at Q = 40). The results also indicate that the difference between the three policies is least when there are small lot sizes and high utilization.

For all factors and all policies, the customer inventory increases as the factor levels increase, although the increase is relatively minor for the utilization factor. Lot size and fill rate have the largest impact on customer inventory; utilization has the least. It is somewhat surprising that the change in customer inventory is relatively small as utilization increases. However, it could be that the factor levels for utilization are low enough that not much effect is observed.

For the fill rate and lead time factors, the manufacturer’s inventory stays constant in the MTO and MTS policies. This occurs because the proportion of time the manufacturer spends producing (the utilization) is fixed, and during the time the manufacturer is not producing it always holds either finished goods inventory (MTS) or no inventory at all (MTO). Because neither the fill rate nor the lead time affect the proportion of time the manufacturer produces or the amount of inventory held when it is not producing, the manufacturer’s inventory is constant for these factors regardless of the factor level.

For an MTO strategy, the manufacturer inventory increases with increasing lot size and utilization. An MTS policy also has increased manufacturer inventory with higher lot sizes, although the manufacturer inventory decreases as utilization increases.
This is explained by noting that the duration for which the finished goods inventory must be carried decreases as utilization increases.

Under VMI, the manufacturer inventory increases only with increased lot sizes and stays relatively constant with increased lead time. Two unexpected results were found in VMI’s results; (1) the relationship between the manufacturer’s inventory and capacity utilization appears concave, and (2) the manufacturer inventory decreases with increasing fill rate. An explanation for these results is that, for the 60% and 70% utilization levels and for the 70% and 80% fill rate levels, the value of \( s_2 \) is very close to MTS’s reorder point, \( s \). To compensate for this low value of \( s_2 \), a high production trigger, \( s_1 \), is needed to achieve the desired fill rate. The result is inventory levels similar to those of an MTS manufacturer for these factor levels.

### 3.5 Serial Supply Chain Simulation

An arrival rate of \( \lambda = 10 \) was used in the numerical analysis, but there are several disadvantages to using an arrival rate this low. Recall that values of \( s \), \( s_1 \) and \( s_2 \) are calculated at specific fill rate levels. At small values of \( \lambda \), it is difficult to achieve the desired fill rate precisely because the Poisson distribution is a discrete distribution. Consider, for example, an MTS environment having \( \lambda = 10 \). If \( s = 10 \), the fill rate may equal 87% while a fill rate of 92% may be achieved if \( s = 11 \). Note that if a fill rate of 90% were desired, the closest that could be achieved would be a fill rate of 92%. At higher levels of \( \lambda \), this is not problematic, as moving from \( s = 100 \) to \( s = 101 \) will yield only a small change in fill rate. Thus, it is possible to obtain fill rate more precisely as \( \lambda \) is increased.
It is also desirable to determine the effect that demand variation has on the relative performance of the three replenishment strategies. The probability calculations use exponential interarrival times having an arrival rate of \( \lambda \). Hence, the number of arrivals over a given time period, \( t \), is Poisson distributed with mean \( \lambda t \). An unattractive feature of the Poisson distribution is the relationship between the variance and the mean. In the Poisson distribution, \( \mu = \lambda \) and \( \sigma^2 = \lambda \), resulting in a coefficient of variation equal to \( \frac{\lambda^{0.5}}{\lambda} \) (Cv = \( \lambda^{-0.5} \)). Thus, as the arrival rate becomes relatively high, the coefficient of variation becomes relatively small and cannot be increased without reducing the arrival rate.

**Simulated Interarrival Distribution**

Because it is desirable to have both an adequately large arrival rate and the ability to control the demand variation, the simulation draws interarrival times from the gamma distribution rather than the exponential distribution. The gamma probability density function, mean, and variance are as follows.

\[
P(X = x) = \frac{\lambda^\alpha \cdot e^{-\lambda \cdot x} \cdot (\lambda \cdot x)^{\alpha - 1}}{\Gamma(\alpha)}
\]

\[
\mu = \frac{\alpha}{\lambda}
\]

\[
\sigma^2 = \frac{\alpha}{\lambda^2}
\]

Where

\( \alpha \) is a shape parameter of the gamma distribution

\( \lambda \) is the rate parameter of the gamma distribution
The gamma distribution was chosen for its relationship with the exponential distribution, which is a special case of the gamma distribution when $\alpha = 1$. To obtain a coefficient of variation larger than that offered by exponential interarrival times, it is necessary for $\alpha < 1$. Of course, it is desirable to calculate the mean, variance, and coefficient of variation of the arrival process when interarrival times are gamma with $\alpha < 1$. This is accomplished in the next section.

**Probability Distribution of $N(t)$ when interarrival distribution is Gamma($\alpha$, $\lambda$)**

Define $N(t)$ as the number of arrivals occurring before time $t$. The probability $N = n$ is

$$P(N(t) = n) = P(N(t) \geq n) - P(N(t) \geq n + 1)$$

The probability that $n$ or more arrivals occur by time period $t$ is equivalent to determining the probability that the time of the $n$th arrival is less than $t$. Let $S_n$ define the arrival time of the $n$th arrival.

$$P(N(t) = n) = P(S_n \leq t) - P(S_{n+1} \leq t)$$

If $X_i$ represent the interarrival time between arrival $i$ and arrival $i-1$, then $S_n = X_1 + X_2 + \ldots + X_n$. Hence, $S_n$ is the n-fold convolution of the interarrival distribution. (Ross 2000). If $X_i$ is Gamma($\alpha$, $\lambda$), then the n-fold convolution of $X$ is Gamma($n \alpha$, $\lambda$). Thus, the probability that $S_n$ is less than or equal to time, $t$, is:

$$P(S_n \leq t) = \int_0^t \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(n \cdot \alpha)} \, dx$$
Where

$$\Gamma(\alpha) = \int_{0}^{\infty} e^{-x} \cdot x^{\alpha-1} \, dx$$

When interarrival times are Gamma ($n^{*}\alpha, \lambda$), the probability distribution for the number of arrivals over time period $t$ is:

$$P(N(t) = n) = \int_{0}^{t} \frac{\lambda \cdot e^{-\lambda x} \cdot (\lambda \cdot x)^{n \cdot \alpha - 1}}{I(n \cdot \alpha)} \, dx - \int_{0}^{t} \frac{\lambda \cdot e^{-\lambda x} \cdot (\lambda \cdot x)^{(n+1) \cdot \alpha - 1}}{I((n + 1) \cdot \alpha)} \, dx$$

To increase the coefficient of variation above that offered by the Poisson distribution while maintaining an arrival rate of 100 arrivals/period, it is necessary to set $\alpha < 1$. However, there is no closed form solution to the gamma function when $\alpha$ is non-integer (Law and Kelton 2000). Thus, to calculate the mean arrivals per period and the standard deviation of the number of arrivals, the results of renewal theory are utilized. Renewal theory holds that, for sufficiently large $t$, the average number of arrivals per period converges to the inverse of the mean interarrival time.

$$\lim_{t \to \infty} \frac{N(t)}{t} = \frac{1}{\mu}$$

The standard deviation of the number of arrivals per period as time approaches infinity converges to:

$$\lim_{t \to \infty} \frac{\text{Var}(N(t))}{t} = \frac{2}{\mu^3}$$

Here, $\sigma$ and $\mu$ are the standard deviation and mean of the interarrival distribution, respectively. Because the simulation uses a Gamma($\alpha$, $\lambda$) interarrival distribution, the limiting mean, variance, and coefficient of variation of the arrival process becomes:
\[ E \left( \frac{N(t)}{t} \right) = \frac{\lambda}{\alpha} \]

\[ Var \left( \frac{N(t)}{t} \right) = \frac{\lambda}{\alpha^2} \]

\[ Cv = \frac{1}{\sqrt{\lambda}} \]

Thus, when gamma interarrival times are used, the coefficient of variation depends solely on the rate parameter and, to increase the variability of the system, the rate parameter must decrease. The mean arrival rate, however, depends on both \( \alpha \) and \( \lambda \). Consequently, the scale parameter can be manipulated to achieve the desired coefficient of variation while maintaining a constant arrival rate.

### 3.5.1 Simulation Parameters and Factors

The discrete-event simulation investigates the same factors as the numeric analysis, but it adds a factor for the demand variation and an additional level for the utilization factor (90%). The factors and their levels are shown in Table 3.5. For each factor level and policy combination, we simulated 5,000 customer replenishments with an arrival rate of 100 units/period. A quadratic search algorithm was utilized to find values of \( s, s_1, \) and \( s_2 \) that meet the desired fill rate.

#### Table 3.6 – Simulation Factor Levels

<table>
<thead>
<tr>
<th>Factor</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot Size</td>
<td>100, 200, 300, 400</td>
</tr>
<tr>
<td>Utilization</td>
<td>50%, 60%, 70%, 80%, 90%</td>
</tr>
<tr>
<td>Lead Time</td>
<td>1, 2, 3, 4</td>
</tr>
<tr>
<td>Fill Rate</td>
<td>70%, 80%, 90%, 98%</td>
</tr>
<tr>
<td>Cv</td>
<td>0.25, 0.5, 0.8, 1.1</td>
</tr>
</tbody>
</table>
3.5.2 Simulation Results

The results of the simulation are used to compare MTO, MTS, and VMI replenishment strategies and determine the impact of the environmental factors in two ways. First, the three replenishment strategies are compared based on the inventory levels obtained at the customer and manufacturer. Next, the three strategies are compared based on the resulting holding costs across the supply chain.

Factor Effect on Manufacturer and Customer Inventory

Figures 3.4 to 3.8 show the average customer and manufacturer inventory for each factor and for each replenishment policy (MTO, MTS, and VMI). The solid line identifies the manufacturer’s inventory, and the customer’s inventory level is denoted with the dashed line. For the most part, the results correspond well with those of the numeric analysis. For example, the manufacturer inventory levels for MTO and MTS remain constant regardless of the fill rate or lead time. Also, the difference in manufacturer’s inventory between the policies increases as lot size increases and is decreases as utilization increases. There are, however, some differences between the results of the numeric analysis and simulation. For example, compared to the numeric analysis, an increase in the lead time, utilization, and fill rate has a much larger impact on the customer inventory for all three replenishment strategies. This is the result of the simulation having higher demand variability than the numerical analysis. This extra variability requires the customer to carry more safety stock.

The results for VMI’s manufacturer inventory is another area where the results between the numerical analysis and the simulation differ. As fill rate is increased, the
manufacturer inventory no longer decreases substantially. With respect to the utilization factor, which displayed a concave relationship in the numeric analysis, VMI demonstrates a linearly increasing function with increasing capacity utilization in the simulation. The simulations’ results differ from those of the numeric analysis because the increase in the arrival rate allows for a more accurate selection of $s_1$ and $s_2$ to meet the fill rate.

Compared to the numeric analysis, the simulation uses an additional factor, coefficient of demand variation, and an additional level for capacity utilization, 90%. The effect of demand variation on inventory levels is shown in Figure 3.7. Similar to other factors, increasing demand variation appears to have little or no effect on the manufacturer’s inventory level, but it causes a large increase in the customer’s inventory. For utilization, the results of the simulation indicate that, as utilization approaches 90%, there is a tremendous effect on the customer’s inventory level. With respect to the manufacturer’s inventory, the three replenishment strategies experience nearly identical levels. This occurs because the manufacturer is producing nearly continuously at this high utilization, giving the manufacturer very little flexibility in timing production and shipments.
Figure 3.4 – Effect of Lead Time on $I_c$ and $I_m$

Figure 3.5 – Effect of Lot Size on $I_c$ and $I_m$

Figure 3.6 – Effect of Fill Rate on $I_c$ and $I_m$

Figure 3.7 – Effect of Demand Variation on $I_c$ and $I_m$
Figure 3.8 – Effect of Utilization on $I_c$ and $I_m$
The effect of the lot size factor also differs between the simulation and the numeric analysis. In the simulation, a convex relationship exists between lot size and customer inventory because of the interaction between lot size and demand variation shown in Figure 3.9. At low levels of demand variability, such as that used in the numeric analysis, customer inventory increases nearly linearly with increasing lot size. At higher levels of demand variability, however, the relationship between the customer’s inventory level and lot size becomes convex in shape.

*Figure 3.9 – Interaction between lot size and demand variation* 

![Graph showing interaction between lot size and demand variation.](image)

To explain the cause of the interaction, Figure 3.10 displays the components of customer inventory, i.e., the cycle stock and safety stock, at different lot sizes. The figure demonstrates that the level of safety stock needed to maintain a constant fill rate decreases as the lot size increases. At lower lot sizes, the reduction in safety stock is greater than the increase in the cycle stock, which causes the average customer inventory to decrease. However, the increase in cycle stock eventually surpasses the decrease in safety stock, causing the customer inventory to increase. This causes the average
customer inventory to display a convex pattern. The pattern is less prominent when the coefficient of variation is low because safety stock comprises a small percent of the customer’s total inventory. Thus, the relationship between customer inventory and lot size closely matches that of the cycle stock and not the safety stock.

*Figure 3.10 – Disaggregation of $I_c$ at Different Lot Sizes*

![Graph showing disaggregation of inventory levels at different lot sizes.](image)

**Factor Effect on Cost**

In keeping with the literature, which predominantly investigates MTS systems (Sahin and Robinson 2005), we employ the MTS strategy as a base case and then calculate the benefit of VMI and MTO as a percent reduction in supply chain holding cost from the MTS system. For each policy, the supply chain holding cost is the sum of the manufacturer’s and customer’s holding costs. We assume holding costs are linear. Consequently, Figures 3.11 -3.20 display the percent cost improvement that MTO and VMI policies offer over an MTS replenishment policy. The x-axis in each figure is the holding cost ratio ($h_c / h_m$), where $h_c$ and $h_m$ represent the customer’s and manufacturer’s
holding costs, respectively. As the ratio increases, the cost of holding inventory at the customer becomes more expensive relative to holding inventory at the manufacturer.

The results indicate that, at lower holding cost ratios, both MTO and VMI yield their largest cost improvement, with MTO offering a larger improvement than VMI. At the lowest cost ratio, $h_c / h_m = 0.5$, MTO offers approximately a 20% improvement over MTS whereas VMI offers close to a 15% cost reduction for most of the environmental factors explored in this study.

For both VMI and MTO, the percent cost improvements generally decline as the cost ratio increases. Furthermore, the rate of decline is lower with VMI than with MTO. Consequently, even though MTO begins with a higher initial benefit at the lowest cost ratios, at some point the benefit of using VMI surpasses that of using MTO. For example, at $h_c / h_m = 2$, all lead time factor levels for both VMI and MTO offer around a 5% cost reduction over MTS. However, as the cost ratio increases, VMI’s cost reduction does not decrease as rapidly as MTO’s and at $h_c / h_m > 2$, VMI offers lower costs than MTO.

As shipping lead times increase, the improvement MTO and VMI offer over MTS increases as well. It should be noted, however, that in the shipping lead time have minimal impact on the VMI and MTO cost reductions at low holding cost ratios changes (Figures 3.11 and 3.12). For example, at $h_c / h_m = 1$, the cost improvement between the four lead time factor levels are practically identical for both MTO and VMI. At higher cost ratios, however, the differences between the lead time factor levels become more pronounced. The trend of increasing value of information for higher levels of lead time is also noted by Chen (1998).
Larger lot sizes also offer a larger improvement for VMI and MTO (Figures 3.13 and 3.14). In contrast to the shipping lead time factor, where the difference in MTO and VMI performance becomes more pronounced as the holding cost ratio increases, for the lot size factor the difference between MTO and VMI becomes smaller at higher cost ratios. In fact, at high enough holding cost ratios, the relationship becomes inverted and higher lot sizes result in lower cost reductions. Chen (1998) finds the same relationship. In addition, Gavirneni et al (1999) note that the benefit of information has a concave relationship with the lot size. Initially, as lot sizes increase, the benefit of information grows. At some point, however, the benefit begins to decrease.

For the remaining factors, MTO and VMI offer the most cost reductions at the lower factor levels. Figures 3.15 and 3.16 demonstrate that lower fill rates offer a larger percentage improvement over MTS for MTO and VMI. For example, when \( \frac{hc}{hm} = 1 \), the cost reduction for MTO is approximately 25% at a 70% fill rate and only 15% at the 98% percent fill rate. For VMI, the difference in the percent cost improvement between the fill rate factor levels is much smaller, especially at low holding cost ratios where each factor results in approximately a 15% improvement.

It appears that the benefit of VMI and MTO is highest at low levels of demand variation (Figures 3.17 and 3.18). Consequently, a small change in inventory level results in a larger percentage improvement. Gavirneni (2002) and Chen (1998) find this same relationship when information is shared with the supplier, although this differs from the relationship found by Fry et al (2001). In that study, the results demonstrated that greater savings occur at high demand variability. An obvious reason for this result, however, is that their model considered penalty costs when the manufacturer exceeded a
minimum (z) or a maximum (Z) inventory level. As the demand variation is increased in their model, the authors simultaneously increased the range (Z – z). Because the range is increased, it is easier to meet the target without suffering penalty costs, thereby increasing the benefit at higher levels of variability.

Utilization has a large effect on the relative performance of MTO and VMI, with the greatest cost savings coming at low utilizations (Figures 3.19 and 3.20). At very high utilizations there appears to be only a small cost advantage for MTO and VMI. Also of note is that, with the exception of the 90% utilization level, MTO and VMI offer lower costs than MTS even for very high levels of holding cost ratio. On the other hand, at a 90% utilization, MTS outperforms MTO and VMI for all but the lowest cost ratio. For a constant demand rate, having a low utilization is analogous to having a high capacity. Consequently, these results support the findings of Gavirneni et al (1999) who determined that the benefit of information sharing increases with increased capacity.
Figure 3.11 – Effect of Lead Time on MTO Cost Performance

Figure 3.12 – Effect of Lead Time on VMI Cost Performance

Figure 3.13 – Effect of Lot Size on MTO Cost Performance

Figure 3.14 – Effect of Lot Size on VMI Cost Performance
Figure 3.15 – Effect of Fill Rate on MTO Cost Performance

Figure 3.16 – Effect of Fill Rate on VMI Cost Performance

Figure 3.17 – Effect of Variation on MTO Cost Performance

Figure 3.18 – Effect of Variation VMI Cost Performance
Figure 3.19 – Effect of Utilization on MTO Cost Performance

Figure 3.20 – Effect of Utilization on VMI Cost Performance
Manufacturer minimizes supply chain costs

Figures 3.11 to 3.20 demonstrate that choice of a replenishment strategy depends on the holding cost ratio. The best policy under certain environment is the one that offers the low cost solution to the following objective function:

$$\text{Min: } (h_c/h_m)I_c + I_m$$

This follows since the average customer and manufacturer inventory levels that result for each policy - MTO, MTS, and VMI – form an efficient frontier, similar to that shown in Figure 3.21. The slopes of the line segments connecting the efficient points represent the indifference levels between the two points joined by the line. For example, if the slope between MTO and MTS is 5, then the total supply chain cost of MTO and MTS is equal at a holding cost ratio of 5. We define this ratio as the indifference ratio. To determine which policy to use under certain environmental conditions, it is only necessary to know the indifference levels for this condition. For example, in Figure 3.21, VMI offers the lowest cost if $21 \geq (h_c/h_m) \geq 3$. If $h_c/h_m < 3$ then MTO is the appropriate policy while if $h_c/h_m > 21$, MTS would offer the minimum cost.
Because a VMI manufacturer is able to time production and shipments, there are multiple VMI policies. In effect, using the shipping and production timing flexibility under VMI, multiple efficient points along a frontier can be created. Thus, VMI’s true benefit is that, at intermediate holding cost ratios, it can offer lower costs than either an MTO or MTS policy. To demonstrate this, refer to Table 3.7, which displays the reduction in supply chain cost VMI offers over the lowest cost that MTO and MTS offer at the indifference ratio. For example, in one environment, MTO results in 33.6 units of customer inventory and 30.1 units of manufacturer inventory. In the same environment, the numbers for MTS are 30.8 and 69.9. This results in an indifference ratio of 14.5 at which the total cost for MTO and MTS are:

MTO: 33.6(14.5) + 30.1 = $516

MTS: 30.8(14.5) + 69.9 = $516
In the same environment, VMI offers inventory levels of 30.9 and 48.0 at the customer and manufacturer, respectively. At a cost ratio of 14.5, this results in a cost of $495 and a percent cost reduction of 4.1%.

Table 3.7 – Potential VMI cost reduction at indifference ratio

<table>
<thead>
<tr>
<th>Lead Time</th>
<th>Cost Improvement</th>
<th>Lot Size</th>
<th>Cost Improvement</th>
<th>Cv</th>
<th>Cost Improvement</th>
<th>Fill Rate</th>
<th>Cost Improvement</th>
<th>Utilization</th>
<th>Cost Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.02%</td>
<td>100</td>
<td>0.91%</td>
<td>0.25</td>
<td>2.08%</td>
<td>70%</td>
<td>1.72%</td>
<td>60%</td>
<td>2.54%</td>
</tr>
<tr>
<td>2</td>
<td>1.76%</td>
<td>200</td>
<td>1.62%</td>
<td>0.5</td>
<td>2.15%</td>
<td>80%</td>
<td>1.79%</td>
<td>70%</td>
<td>1.81%</td>
</tr>
<tr>
<td>3</td>
<td>1.57%</td>
<td>300</td>
<td>1.99%</td>
<td>0.75</td>
<td>1.62%</td>
<td>90%</td>
<td>1.85%</td>
<td>80%</td>
<td>0.93%</td>
</tr>
<tr>
<td>4</td>
<td>1.46%</td>
<td>400</td>
<td>2.29%</td>
<td>1</td>
<td>0.96%</td>
<td>98%</td>
<td>1.46%</td>
<td>90%</td>
<td>0.22%</td>
</tr>
</tbody>
</table>

Rather than displaying the percent improvement of VMI over a base strategy, Table 3.7 shows the potential improvement offered by VMI over both MTO and MTS strategies. The results demonstrate VMI offers the most potential for reducing cost when lead time, variation, and utilization are low, lot size is high, and at moderate fill rates. As was the case for previous results, utilization and demand variation have the largest impact on the performance of VMI.

3.6 Results Discussion

One of the primary objectives of this research was to determine the effect several environmental variables have on the MTO, MTS, and VMI replenishment strategies. We accomplished this by comparing both inventory data and cost performance under various fill rates, lead times, utilizations, lot sizes, and levels of demand variation. The results indicate that the lead time factor creates only small increases in the customer’s inventory level and no change in the manufacturer’s inventory. It also has the least effect on the relative benefit of MTO and VMI systems compared to MTS systems.
Changes in demand variation and utilization have the largest impact on customer inventory levels for all replenishment strategies, particularly with respect to the customer’s inventory levels. For the utilization factor, this is especially true at levels above 80%. These same two factors also have the largest impact on the relative benefit of VMI and MTO strategies over an MTS system and for the benefit of VMI compared to both MTS and MTO systems at the indifference cost ratio.

Another objective of this research was to determine how VMI compared in performance to an MTO strategy. From the inventory results, it is obvious that MTO offers the lowest inventory levels at the manufacturer, although it does result in the highest inventory levels at the customer. Consequently, MTO is the appropriate pick for environments having a low \( h_c/h_m \) ratio. At the lowest cost ratios, MTO and VMI offer approximately 20% and 15% cost reductions, respectively, compared to MTS. As the cost ratio increases, however, MTS becomes more tenable, eventually offering a lower supply chain cost than MTO.

The final objective of this study was to determine if VMI’s flexibility in production and shipment could offer benefit over MTS and MTO, based purely on production and shipping timings and without relying on shipping consolidations, reduced lot sizes, allocation of inventory, and risk pooling for improvements. The results of Table 3.7 indicate that VMI can indeed offer improved costs based solely on improved production and shipment timing. In effect, VMI’s benefit is that it can be utilized to manipulate the manufacturer and customer inventories in a way that corresponds with the holding cost ratio. By doing so, the total supply chain costs are minimized. Furthermore,
it should be noted that neither MTO nor MTS replenishment policies can be used to manipulate the inventory levels in this manner.

It is important to note that VMI can at worst offer no improvement over either an MTO or MTS strategy. This is because the flexibility in production timing and shipment afforded to VMI enables it to replicate these other policies. In other words, if the holding cost ratio is especially low, VMI can be implemented in a manner comparable to MTO. Likewise, if the holding cost ratio is especially high, VMI can be implemented in a manner equivalent to MTS. Thus, VMI may be most beneficial in environments where the current replenishment strategy does not reflect for true holding cost ratio. A move to VMI allows the manufacturer to choose the proper inventory levels across the supply chain for the holding cost ratio. For example, if the manufacturer currently employs an MTS strategy in a low holding cost ratio environment, a move to VMI would allow the manufacturer to operate in a manner more similar to MTO. As the results in this section indicate, this could result in a cost reduction of approximately 10% to 20%.
4. VMI PRODUCTION BENEFIT IN AN ARBORESCENT SUPPLY CHAIN

This chapter addresses VMI’s benefit in a supply chain where multiple customers each demand a product which only they demand. Thus, inventory for one customer cannot be used to satisfy demand for another customer. We study an environment where the vendor is a manufacturer. As such, much of the potential benefit of VMI derives from the manufacturer’s ability to improve production timing. Three methods of timing replenishments are investigated. These include a policy based on each customer’s individual inventory level, another based on an aggregate measure of the total customer inventory, and also a heuristic policy. The heuristic policy works by calculating for each customer their expected fill rate assuming immediate production. Production is initiated when this value reaches a predetermined level. Results indicate the heuristic works best in environments where a low fill rate is used and where the manufacturer has a high utilization.

4.1 Introduction

Under VMI, the manufacturer has complete authority over the timing of replenishment shipments to the customer. The information sharing found in a VMI program makes it possible to better manage production (Zheng and Zipkin 1990), although manufacturers do not always make use of this information in their manufacturing planning systems (Schenck and McInerney 1998).

When a manufacturer controls replenishments for a single product and a single customer, it can choose a specific reorder point to yield the desired service level. When a VMI manufacturer produces a single product for multiple customers, a popular approach...
is for the manufacturer to choose a reorder point based on the total customer inventory. Under this approach, once the total inventory across all customers reaches a reorder point, the manufacturer allocates the available stock between the customers so that each customer’s inventory position is brought to the same service level (Clark and Scarf 1960).

In practice, it is not uncommon for a manufacturer to produce multiple products on a single machine (Vergin and Lee 1974). This complicates the manufacturer’s production timing decision, particularly in situations where the products share common manufacturing resources and co-production is impossible. In this situation, a problem arises for a VMI manufacturer in that each customer demands a different product, and one product cannot be substituted for another, making it infeasible to allocate inventory across customers. Consequently, customers cannot share the risk of a shortage between them, as is done under an allocation scheme, and thus some of the benefit of a VMI program appears to be lost.

Because all products share a common production facility, their performance is heavily interdependent. Thus, the management of production is a critical issue. Implementation of a VMI program may help firms in such an environment. By observing demand at all customer sites, the manufacturer is made conscious of instances where the customers’ needs for production time interfere with one another. Given a VMI environment, an important objective is to determine the best production control policy.

To study this issue, we investigate VMI’s benefit in an arborescent supply chain where each customer orders a unique product and the manufacturer can produce only one product at a time. Each period, the manufacturer inspects the customers’ inventory positions and either begins producing a batch for one of the customers or remains idle.
Lot sizes are exogenous and predetermined. The need to produce a batch of size $Q$ may stem from such factors as a desire to ship full truckloads or the existence of ordering costs (Zipkin 1986). We examine three models for making the production decision, (1) a base case where the manufacturer sets individual reorder points for each customer, (2) the manufacturer has a single reorder point based on the aggregate customer inventory, and (3) a heuristic policy that initiates production when the expected fill rate reaches a threshold level.

The next section discusses these different models and the logic supporting their use. Following this is a description of the supply chain. Subsequently, we describe a numeric analysis and present the results. The results section is further divided into two sections; one section discusses the effect of the environmental factors on supply chain performance and the other section compares the models’ performance to one another.

### 4.2 Supply Chain Description

This research investigates a two-echelon arborescent supply chain where a manufacturer replenishes two customers. In this supply chain, the manufacturer has access to each customer’s inventory information and acts as a central decision maker in the supply chain. Each customer requires a unique product and hence inventory cannot be shared between them. In case a customer has insufficient inventory to cover demand, excess demand is backordered.

Unlike the first study, where continuous review inventory policies were considered, here the manufacturer checks inventory periodically. Demand during a period follows a gamma distribution and the customers’ demand distributions need not be
identical. Each period begins with the arrival of the shipments sent to the customers in the previous period. Hence, there is a one-period shipping lead time. The arrival of shipments is followed by the completion of any production batch scheduled to finish in that period. Whenever production is complete, the manufacturer immediately ships the completed batch to the appropriate customer. If the manufacturer is idle, the decision of whether to produce in the current period is then made using one of the three models put forth in this study. No setup time is incurred at the outset of production. Finally, demand occurs at the customers’ sites and all performance measures are calculated at the end of the period.

The manufacturer can produce a fixed quantity, C, each period and this capacity holds for either product. A batch for customer c is defined as Q_c and the size of production batches are based on a time supply of demand for each customer. We base lot sizes on a time supply of demand with the intention of creating a production sequence that naturally alternates between customers and causing the number of production runs to be relatively equal for all customers. As a result, the customers should require replenishments at different times and hence reduce interference between customers’ production needs at the manufacturer. In case the manufacturing time needed to produce Q_c units does not equal an integer number of periods, the lot size is increased to a point where production time equals an integer number of periods, as in Graves (1980). As a result, the customers carry some remnant inventory, similar to the case under MRP systems. The reason for increasing lot sizes in this manner is to ensure that the manufacturer is fully utilized in any production period. If the manufacturer were not, then problems could arise in high utilization environments.
The manufacturer carries no finished goods inventory; hence, when a batch of $Q_c$ units is complete, the entire batch is immediately shipped to customer $c$. The customer receives a shipment one period after it is sent. Thus, the total replenishment lead time, assuming the manufacturer is idle, is $Q_c/C + 1$. As demonstrated in the first study, by replenishing customers in this manner the manufacturer minimizes both its own and the total supply chain inventory.

The problem considered in this study shares several features with the Stochastic Economic Lot Scheduling Problem (SLSP). The SLSP itself is based on the Economic Lot Scheduling Problem (ELSP) whereby multiple products having fixed demand rates are scheduled on a single machine having limited capacity. Solving the ELSP problem involves creating a cycle schedule whereby items are manufactured in a predetermined sequence ad infinitum (Federgruen and Katalan, 1996). Under the ESLP, backordering is not allowed and the objective is to minimize long-run setup and holding costs.

The key difference between the ELSP and the SLSP is the introduction of random demand and the existence of backorders. Because calculation of an optimal policy is extremely difficult, heuristics are prevalent in the literature. Under ESLP, a static cycle is created and the manufacturing sequence of products is fixed within a cycle. SLSP also differs from ESLP because many of the heuristics used for scheduling allow for dynamic production sequencing. In this case, the manufacturer makes production decisions based on the system-wide inventory levels and is not constrained to a predetermined sequence.

Although the problem setting considered in this study is similar to that of the SLSP problem, the assumptions used differ in several ways. Primarily, this study considers fixed lot sizes corresponding to a time supply of demand. We use equal time
supplies so that the production sequence tends to alternate between the customers' products and thereby minimizes machine congestion. Under SLSP, order sizes vary with each production run. Another important distinction is that the VMI manufacturer's objective in our study is to minimize the supply chain inventory while meeting an overall fill rate target, whereas SLSP allows the customer service level to fluctuate based on cost considerations.

4.3 Production Models

In Model 1, a base case, the VMI manufacturer sets individual reorder points for each customer, producing a batch for a customer when its inventory position is below this level. This is similar to the approach used by Zipkin (1986), who utilized reorder point, order quantity policies for scheduling production. Let $I_c$ represent on-hand inventory for customer $c$ and $O_c$ its on-order inventory. Then production is initiated when $I_c + O_c < R_c$, assuming the manufacturer is idle. If $I_c + O_c < R_c$ for more than one customer, the manufacturer produces for the customer having the shortest run-out time, measured as $(I_c + O_c)/D_c$ where $D_c$ is the mean demand per period of customer $c$.

We also investigated a model identical to Model 1 except that the production decision is made on a First-Come-First-Serve (FCFS) basis. This alternative model represents a non-VMI environment since the manufacturer does not base production decisions on customer inventory levels. We found the performance of these two models to be nearly identical, although the use of shortest run-out time instead of FCFS resulted in an average of 0.56% less inventory. Because the performance of the two models were so similar, an alternative view of Model 1 is that it represents the performance of a
traditional supply chain where customers determine their own reorder levels, there is no information sharing, and there is no centralized control.

Under Model 2, the manufacturer production is based on the total customer inventory level. By basing the production decision on the aggregated inventory rather than individual inventory levels, the manufacturer considers that when the total supply chain inventory is low, the production needs of the customer likely interfere.

In this model, the manufacturer sets an aggregate reorder point, $R_0$, for the entire system. When the total system inventory position drops below $R_0$, production commences for the customer having the shortest run-out time. In other words, if $(I_1 + I_2 + O_1 + O_2) < R_0$ and the manufacturer is idle, then production is commenced for the customer who has the shortest run-out time. In situations where $(I_1 + I_2 + O_1 + O_2) < R_0$ at the end of a production run, the manufacturer will once again produce for the customer with the shortest run-out time.

Model 3 is the heuristic policy. Each period, the heuristic calculates the expected fill rate for each customer assuming production is initiated in the current period. In calculating the fill rate, the manufacturer uses knowledge that the demand over the production and shipping lead time follows a gamma distribution. Here, $\alpha_c$ represents the shape parameter and $\beta_c$ the scale parameter of the demand distribution over the total lead time for customer c. Define $\varphi(IP_c, \alpha_c, \beta_c)$ as the expected shortage for customer c having inventory position $IP_c$, assuming production is begun immediately. Note that the customers can have different shape parameters due to differences in lot sizes and demand rates, thus the necessity of the subscript c.
The heuristic initiates production when one of two situations occur. In the first situation, the manufacturer begins production when it is idle and one of the customer’s expected fill rate is less than or equal to H’.

**Produce when**

\[ 1 - \varphi \left( IP_c, \alpha_c, \beta_c \right)/Q_c \leq H. \]

Where

\[
\varphi \left( IP_c, \alpha_c, \beta_c \right) = \int_{IP_c}^{\infty} \left( x - IP_c \right)^{\alpha_c - 1} \cdot e^{-\beta_c \cdot x} \cdot \frac{\Gamma(\alpha_c)}{\Gamma(\alpha_c)} \, dx
\]

**Fill Rate** = \[ 1 - \frac{\varphi \left( IP_c, \alpha_c, \beta_c \right)}{Q_c} \]

The heuristic also considers interference between customer orders by calculating the expected fill rate for consecutive production runs. Two separate calculations are made, one for the case where the production sequence is customer 1 first and customer 2 second, and a second calculation for the opposite sequence. To determine the fill rate, the average expected shortage for these two consecutive production runs is calculated for each production sequence. Define this function as \( \xi \left( IP_c, \alpha_c, \beta_c \right) \). Let the subscript c identify the customer whose product is manufactured first and let the subscript 0 identify parameters values that result for the customer whose product is manufactured second. Note that for the second production run the demand distribution used in the calculations is based on the total production lead time, \( (Q_1+Q_2)/C \).

\[
\xi \left( IP_c, \alpha_c, \beta_c \right) = \frac{\varphi \left( IP_c, \alpha_c, \beta_c \right) + \varphi \left( IP_0, \alpha_0, \beta_0 \right)}{2}
\]
The second situation in which the manufacturer initiates production for customer c occurs when \( \max [1 - \xi (IP_1, \alpha_1, \beta_1)/Q_1, 1 - \xi (IP_2, \alpha_2, \beta_2)/Q_2] \leq H \). Note that the reorder levels used by the heuristic when deciding to produce in the two situations, H and H’, need not be the same value. Furthermore, it should be understood that the manufacturer is not required to produce the second production run immediately upon completing the first if this customer’s inventory position does not warrant it. Also be aware that when there is a large disparity in the demand rates between the two customers, instances may arise where both \( \varphi (IP, \alpha, \beta) \leq H \) and \( \max [\xi (IP_1, \alpha_1, \beta_1), \xi (IP_2, \alpha_2, \beta_2)] \leq H’ \) call for a production to begin, however, each calls for different product to be produced. In these cases the product called for by \( \max [\xi (IP_1, \alpha_1, \beta_1), \xi (IP_2, \alpha_2, \beta_2)] \leq H’ \) is produced first.

To understand how the heuristic minimizes inventory, consider that there is a nonlinear relationship between fill rate and average inventory. As the amount of inventory held increases, the marginal benefit to the fill rate decreases. Thus, to reach a desired service level, as is the objective in this study, it is best to produce at an inventory level that yields an expected fill rate that exactly matches the desired fill rate.

For example, consider the fill rate and inventory levels averaged across two production cycles. Assume there is a level of inventory, \( I^* \), so that when an order is placed at \( I^* \), an expected fill rate, \( F^* \), is obtained. If \( F^* \) equals the desired long-run fill rate, then to obtain it, the manufacturer may initiate production for both cycles when the available inventory, I, is exactly equal to \( I^* \) in both cases. This implies that the expected fill rate for both production cycles is equal. Conversely, the manufacturer may attempt to meet the fill rate target by initiating the first production run when the current inventory is
at a point, $I_1$, which is less than $I^*$ and the second production run at a point, $I_2$, which is greater than $I^*$.

Assume the lead time is the same for all production runs. Then in the first situation, where production is initiated for both cycles when $I = I^*$, the average ending inventory across both runs is simply $I^* - E[x]$ where $E[x]$ is the expected demand during the lead time. For the second situation to provide an equal amount of inventory, $(I_1 + I_2)/2$ must equal $I^*$. However, by initiating production at these inventory levels, the average fill rate of the two cycles, $F$, will be lower than the target fill rate, $F^*$, since the relationship between inventory and fill rate is non-linear, as shown in Figure 4.1. This implies that for two production cycles, if production is initiated at $I_1 < I^*$, then for $F$ to equal $F^*$, the average inventory, $(I_1+I_2)/2$ will be larger than $I^*$. Consequently, average inventory is minimized when production is initiated at the time the expected fill rate is equal to the desired fill rate, $F^*$.

*Figure 4.1 – Relationship between inventory and fill rate*
Figure 4.2 shows a contour plot of the function \( \max [1-\xi (IP_1, \alpha_1, \beta_1)/Q_1, 1-\xi (IP_2, \alpha_2, \beta_2)/Q_2] \) at different levels of inventory for customers 1 and 2 in a situation where both customers are identical. In this plot, the curved lines depict points having the same fill rate with the upper right corner of the chart having the highest fill rate because both customers’ inventory levels are highest at this point. Because closely matching the desired fill rate reduces the inventory needed to reach that fill rate, the relative benefit of the three models should depend on how closely they approximate the respective contour plot for a given set of factor levels. The more closely a model approximates the contour plot (which obviously takes on different shapes depending on the factor level settings), the lower the inventory that must be carried by the customers.

Figure 4.2 – Contour Plot of \( \max [1-\xi (IP_1, \alpha_1, \beta_1)/Q_1, 1-\xi (IP_2, \alpha_2, \beta_2)/Q_2] \)
4.4 Arborescent Supply Chain Simulation

4.4.1 Factor Levels

To test the performance of the three strategies, we developed a simulation of the supply chain using Microsoft Visual Basic for Applications. The simulation records the resulting fill rate and inventory levels for each model. A list of these factors and their levels can be found in Table 4.1. Note that we set the shipping lead time equal to a single period.

Table 4.1 –Environmental Factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot Sizes</td>
<td>4, 8, 12, 16 periods of supply</td>
</tr>
<tr>
<td>Coefficient of Demand Variation</td>
<td>0.5, 0.75, 1.0, 1.5</td>
</tr>
<tr>
<td>Production Capacity</td>
<td>50, 75, 100, 150 units/period</td>
</tr>
<tr>
<td>Demand Split</td>
<td>25%-75%, 32.5%-62.5%, 45%-55%, 50%-50%</td>
</tr>
<tr>
<td>Fill Rate</td>
<td>70%, 80%, 90%, 98%</td>
</tr>
</tbody>
</table>

We consider lot sizes to be an exogenous variable, as in Graves (1980), and model the case where the manufacturer produces a certain time supply of demand in each production batch. A manufacturer produces in batches when there are fixed shipping capacity or setup and ordering costs. Because we base lot size on a time supply of demand, we calculate lot sizes as demand rate multiplied by the periods of supply for which the manufacturer wants to produce. Note that basing lot sizes on periods-of-supply factor only partially determines the actual lot size because the customer’s demand rate also has an impact. For example, if a customer has a demand rate of 10 units/period and the manufacturer wishes to produce 4 days of supply during each production run, then a production batch for this particular customer equals 40 units.

Across all factor settings, both customers have a combined demand rate of 40 units/period. The demand split factor then determines each customer’s individual
demand rate for a particular simulation run. The four demand split percentages translate into demand rates of 10, 12, 15, and 20 units/period for customer 1 and demand rates of 30, 18, 25, and 20 units/period for customer 2. These are always grouped as (10, 30), (12, 18), (15, 25), and (20, 20) for the simulations.

In each period, the simulation draws the random demand variable from a gamma distribution. To control the coefficient of variation of demand, it is necessary to manipulate the shape parameter, $\alpha$. Four levels for the coefficient of variation, $Cv$, for a single period are investigated with each corresponding to a different level for $\alpha$. These are ($Cv = 0.5, \alpha = 4.00$), ($Cv = 0.75, \alpha = 1.78$), ($Cv = 1.0, \alpha = 1.00$), and ($Cv = 1.5, \alpha = 0.44$).

For each period, the manufacturer chooses to produce, it produces at capacity. Utilization rates thus correspond to the percentage of period in which production occurs. The corresponding utilizations for the above production capacity are ($C = 50, \text{Utilization} = 80\%$), ($C = 75, \text{Utilization} = 53.3\%$), ($C = 100, \text{Utilization} = 40\%$), ($C = 150, \text{Utilization} = 26.7\%$).

For each of the 1,024 factor-level combinations, a quadratic search determines the values of $R_c$ (Model 1), $R_0$ (Model 2), and $H'$ (Model 3) needed to reach the desired fill rate for all settings. The extant literature commonly utilizes search procedures due to the complexity of the problem. For example, Federgruen and Katalan (1998) utilize a Golden Section search in their study. In this study the quadratic search finds the reorder levels for each model that minimize the function $(FR - FR^*)^2$, where $FR$ is the fill rate at the current (non-optimal) solution and $FR^*$ is the desired fill rate. The relationship between $FR$ and each model’s reorder level is predictable, i.e., an increase in the reorder
level corresponds to an increase in the fill rate. Because the function \((\text{FR} - \text{FR}^*)^2\) simply subtracts a constant and squares the result, it too is well behaved.

We simulate a total of 2,500 customer replenishments for each step in the search to estimate the fill rate using a common random number seed. After the quadratic search has determined the reorder levels for each model, we use the reorder levels to collect data in 10 simulations of 2,000 total replenishments for each factor level. In this case, each simulation utilizes a different random number seed.

### 4.4.2 Simulation Verification

One method of verifying the simulation is to trace the events over a period of time, ensuring that events occur as expected (Law and Kelton, 2000). In addition to tracing the simulation manually, creating charts of key measures is also helpful. Figures 4.2 to 4.4 plot the decision criteria used by Models 1, 2, and 3 to make their respective production decisions (primary y-axis) from period 250 to period 400 (x-axis). The secondary axis, which is reverse scaled, displays the manufacturer’s inventory level.

Because Model 1 uses each individual customer’s inventory level to time production, Figure 4.2 displays both customers’ inventory on the y-axis. Similarly, because Model 2 bases the production decision on the aggregate inventory level, Figure 4.3 plots the aggregate inventory level and the production sequence when this level reaches reorder point \(R_0\). Finally, Figure 4.4 displays the fill rate function, \(\max [1-\xi (\text{IP}_1, \alpha_1, \beta_1)/Q_1, 1-\xi (\text{IP}_2, \alpha_2, \beta_2)/Q_2]\), and its resulting production pattern. Figures 4.5 to 4.7 trace the customer inventory levels through the same timeframe as that presented in
Figures 4.2 to 4.4. Together, these figures help verify that production is initiated as intended by each replenishment model and that inventories are calculated correctly.
Figure 4.2 – Model 1 Production Decision Variables and Production Timing

Figure 4.3 – Model 2 Production Decision Variables and Production Timing

Figure 4.4 – Model 3 Production Decision Variable and Production Timing
Figure 4.5 – Model 1 – Customer on-hand inventory levels

Figure 4.6 – Model 2 – Customer on-hand inventory levels

Figure 4.7 – Model 3 – Customer on-hand inventory levels
4.5 Simulation Results

4.5.1 Factor Effects on Average Inventory

Table 4.2 displays the results for each environmental factor. Because the manufacturer follows a MTO strategy under each of the three models, the manufacturer’s inventory is identical between them. Thus, the three models are compared using the total customer inventory, summed across all customers.

The results indicate that, regardless of policy, an increase in demand variance increases the customers’ need to carry inventory. This increase derives from the need for the customers to carry additional safety stock to guard against the increased uncertainty. The results in Table 4.2 also demonstrate that average inventory levels increase when the manufacturer increases the days of supply produced in each production batch. Although increased cycle stock levels cause some increase in customer inventory, increasing production lead times and the corresponding increase in safety stock levels also contribute to the higher levels of customer inventory.

As the difference in the customers’ demand rates increases, the expectation is that the customer inventory would decrease. This follows because the system performance approaches that of a single-product system as one customer’s demand rate becomes much larger than the other customer’s. However, the results reveal that the average inventory is higher at both the highest and lowest factor levels than at the intermediate factor levels. As will be discussed shortly, this is a result of the manner in which lot sizes are calculated.

Similar to the demand split factor, the production capacity factor has its highest average inventory at the extreme factor levels. At the highest level of production
capacity, the time to produce a lot should be lower than at other factor levels, causing the production lead time to be shorter and therefore requiring less safety stock. Thus, there is an expectation that the average inventory should be smallest at the highest capacity level. For factor levels C = 50, 75, and 100 units/period, the results hold true to the expected relationship; only the C = 150 factor level offers a counterintuitive result.

Table 4.2 – Factor Effects across Levels: Average Inventory

<table>
<thead>
<tr>
<th>Demand Variance (Cv)</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>168.2</td>
<td>184.2</td>
<td>209.7</td>
<td>306.2</td>
</tr>
<tr>
<td>Model 2</td>
<td>184.6</td>
<td>202.0</td>
<td>226.8</td>
<td>322.9</td>
</tr>
<tr>
<td>Model 3</td>
<td>166.5</td>
<td>183.0</td>
<td>207.5</td>
<td>301.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand Split</th>
<th>25% - 75%</th>
<th>32.5% - 62.5%</th>
<th>45% - 55%</th>
<th>50% - 50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>223.4</td>
<td>212.4</td>
<td>209.2</td>
<td>223.2</td>
</tr>
<tr>
<td>Model 2</td>
<td>239.6</td>
<td>229.1</td>
<td>228.7</td>
<td>239.0</td>
</tr>
<tr>
<td>Model 3</td>
<td>215.2</td>
<td>210.5</td>
<td>208.3</td>
<td>224.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Production Capacity</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>233.7</td>
<td>210.8</td>
<td>204.2</td>
<td>219.6</td>
</tr>
<tr>
<td>Model 2</td>
<td>233.8</td>
<td>226.4</td>
<td>225.1</td>
<td>251.1</td>
</tr>
<tr>
<td>Model 3</td>
<td>226.8</td>
<td>209.0</td>
<td>203.3</td>
<td>219.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lot Size (Periods of Supply)</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>140.3</td>
<td>191.0</td>
<td>237.5</td>
<td>299.4</td>
</tr>
<tr>
<td>Model 2</td>
<td>148.8</td>
<td>207.4</td>
<td>257.6</td>
<td>322.6</td>
</tr>
<tr>
<td>Model 3</td>
<td>138.4</td>
<td>189.3</td>
<td>235.3</td>
<td>295.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fill Rate</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>98%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>103.5</td>
<td>160.6</td>
<td>235.0</td>
<td>369.1</td>
</tr>
<tr>
<td>Model 2</td>
<td>107.7</td>
<td>169.7</td>
<td>254.4</td>
<td>404.6</td>
</tr>
<tr>
<td>Model 3</td>
<td>99.1</td>
<td>158.3</td>
<td>234.8</td>
<td>366.0</td>
</tr>
</tbody>
</table>

For each combination of production capacity and demand split factor levels, Table 4.3 averages the lot sizes used (based on periods of supply). As can be seen, the lot sizes
vary somewhat between the different factor levels. In this case, the lot size actually increases with increasing production capacity. This occurs because the quantity of remnant inventory is highest at higher production capacity. This explains the results observed in Table 4.2 for the production capacity factor. Similarly, the simulation results found in Table 4.2 reflect the relationship between the average lot sizes and the demand split factors shown in Table 4.3. Consequently, the results when production capacity is 150 units/period and when the demand split factor is at either its highest or lowest and highest levels may be artifacts of the method used to determine lot sizes.

Table 4.3 – Customer Inventory at Capacity and Demand Split settings

<table>
<thead>
<tr>
<th>Split</th>
<th>Capacity (units/period)</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50</td>
<td>75</td>
</tr>
<tr>
<td>10</td>
<td>225.0</td>
<td>243.8</td>
</tr>
<tr>
<td>15</td>
<td>212.5</td>
<td>234.4</td>
</tr>
<tr>
<td>18</td>
<td>225.0</td>
<td>225.0</td>
</tr>
<tr>
<td>20</td>
<td>225.0</td>
<td>262.5</td>
</tr>
<tr>
<td>Average</td>
<td>221.9</td>
<td>241.4</td>
</tr>
</tbody>
</table>

4.5.2 Comparison of Models

If two separate production models yield nearly identical production patterns, (i.e., under each model, production for the same customer is initiated at the similar levels of customer inventory), then the resulting fill rate and average inventory levels would be nearly equal. However, as the production pattern between the two policies grows increasingly different, then so too should their performance.

Figures 4.8 to 4.11 display the customer inventory levels at the time production runs are initiated and demonstrate how the models’ ordering patterns are very different. Model 1 utilizes two reorder points, one for each customer, and thus it has a grouping of orders on or near these reorder points, creating an L-shaped ordering pattern (see Figure
Model 2 utilizes a single reorder level that is based on the aggregate inventory, so it will always have a grouping of orders along a line yielding a constant sum of customers 1 and 2’s inventory levels (see Figure 4.9). Because Model 3 is based on a nonlinear function, its ordering pattern is not necessarily similar across different environments as is the case for Models 1 and 2. In certain environments, this function resembles one that may be obtained from using either Model 1 or Model 2, while in other environments the function will resemble neither.

*Figure 4.8 – Model 1 Orders (Cv = 1.0, C = 50, Fill Rate = 70%, D₁ = 10, D₂ = 30)*
Figure 4.9 – Model 2 Orders (Cv = 1.0, C = 50, Fill Rate = 70%, D₁ = 10, D₂ = 30)

Figure 4.10 – Model 3 Orders (Cv = 1.0, C = 50, Fill Rate = 70%, D₁ = 10, D₂ = 30)
Model 3 inherently considers interference between customer production needs, which is not the case for Model 1. Furthermore, Model 3 prevents either of the customers’ inventory from dropping below a certain level, which is not the case in Model 2. Thus, the difference between these two models rests mainly on the level of interference that is present in the simulation environment in question.

In comparing the three models, Table 4.2 demonstrates that Model 2 performs significantly worse than the others. While this is true on average, Model 2 does offer better performance than Model 1 in certain environments. An analysis of the data shows that when the demand split is in the highest setting (largest difference between customer demand rates) and production capacity is lowest, Model 2 offers consistently better performance than Model 1 for fill rates of 70% and 80%. However, Model 2 never consistently surpasses the performance of Model 3, thereby demonstrating the superiority of Model 3.
The reason Model 2 performs relatively poorly is that it does not consider individual customer inventory levels. Consequently, if its reorder point, $R_0$, equals 10, it does not consider whether customer 1 has zero inventory and customer 2 has 10, or if both customers have 5 units even though it is likely the overall system fill rate will be much lower in the first case. Because Model 2 performs relatively poorly compared to the other models in the majority of factor settings, the remaining discussion is limited to comparisons between Models 1 and 3.

Figures 4.12 to 4.15 display the percent difference in total customer inventory between Models 1 and 3 at the various fill rate levels. In all environments, it appears that the heuristic’s benefit is greatest at lower fill rates and provides very little or even no benefit at higher fill rates. This is explained by noting that the ordering pattern of Model 3 is nearly identical to that of Model 1 at these higher fill rates. Consequently, at higher fill rates Models 1 and 3 initiate production at roughly the same times and therefore they yield similar results.
Fig 4.12 - Model Comparison (Demand Split)

Fig 4.13 Model Comparison (Utilization)

Fig 4.14 - Model Comparison (Days of Supply)

Fig 4.15 - Model Comparison (Cv)
Figure 4.12 shows that the heuristic offers its greatest benefit when there is a large discrepancy between the mean demand rates of the two customers. As the difference in demand rates between the two customers increases, the penalty for not considering the interference between customer orders grows larger. This is partly due to the manufacturer producing a constant quantity each period, regardless of customer.

When one customer has a much lower demand rate than the other, it also has a much shorter production lead time. Due to this short lead time, this customer would not need to carry significant amounts of safety stock. The problem arises, however, that in situations where the production needs of the two customers interfere and the smaller customer waits for the completion of the larger customer’s much longer production run, the smaller customer will experience significant shortages. Conversely, if the production needs of the two customers interfere and the larger customer must wait for completion of the smaller customer’s production run, the fill rate penalty to the larger customer is not as great because the production time of the smaller customer may be much less than its own. Consequently, it is often better in these situations to produce for the smaller customer first.

Thus, as customer demand rates become very different, it is important that manufacturer choose the correct production sequence to minimize the impact on fill rate. Model 3 considers the impact of production sequence on the fill rate, whereas Models 1 and 2 do not. Consider a situation where the larger customer’s inventory causes it to have slightly shorter run-out time than the smaller customer. Models 1 and 2 would both produce for the larger customer first, even though producing for the smaller customer first might in fact offer lower overall fill rates. This problem grows larger as the
difference in customer demand rates increase, which explains why Model 3 offers increasingly better results than Model 2.

The heuristic offers its next highest level of inventory savings over Model 1 in situations where the production capacity is smallest (Figure 4.13). At a 70% fill rate and the lowest level of production capacity, 50 units/period, Model 3 offers 8% less inventory than Model 1. At a 98% fill rate and same production capacity it offers a 1.5% improvement. As production capacity is lowered, the implication is that the manufacturer becomes more highly utilized. As utilization increases, so too will the interference between customer production needs. Because Model 3 controls for interference, and Model 1 does not, Model 3 offers improved performance.

For a given level of the lot size (periods-of-supply) factor (Figure 4.14), the benefit of the heuristic over that of Model 1 remains constant. At a 70% fill rate, the improvement is approximately 4.5%, and there is practically no improvement at the higher fill rate levels. The lot size factor causes increasing amounts of inventory to be held, regardless of the model used. Furthermore, as lot sizes increase at a given fill rate level, so to do the differences in customer inventory between Models 1 and 3. Thus, as in percentage terms the difference stays constant. From the perspective of interference between customer production needs, an increase in the lot size factor does little to affect a difference in interference. In essence, as the lot sizes are increased, the lead time for production increases. However, it increases proportionally for both Models 1 and 3, resulting in the same percent difference.

As demand variation increases, the probability of observing either a very small or very large demand during a period becomes higher. This leads to a greater probability of
the two customers having their production needs interfere. For this reason, Model 3 offers improved timing of production than Model 1. Hence, at higher levels of demand variation and a 70% fill rate, the improvement of Model 3 over Model 1 increases slightly from 4% to 5% as shown in Figure 4.15.

4.6 Results Discussion

Based on information sharing and manufacturer replenishment authority, VMI offers many ways to improve the performance over that of a traditional supply chain. These include consolidation of shipments, inventory allocation, coordination of orders, and improved production timing.

A VMI manufacturer has several models by which it can schedule production. This study investigated several of these models and found that there is considerable value to utilizing more advanced production policies, such as Model 3, especially in certain environments. For example, Model 3 offers a 41.5% improvement over Model 2 in an environment where Cv = 0.75, lots are equal to 4 days of supply, fill rate = 70%, and D₁ = 10 units/period, D₂ = 30 units/period.

As our results show, Model 3 does not offer improved performance in all environments. In situations where D₁ = D₂ = 20, the results between Model 3 and Model 1 are nearly identical in every combination of the other factors levels’ settings. Likewise, when the fill rate is 98%, there are several settings where Model 3 does not surpass the performance of Model 1.

Assuming that a certain value is placed on a production model’s simplicity, there are environments where Model 1 is preferable to Model 3, even though they offer similar
results. Hence, the decision of which production planning model to use very much depends on the supply chain environment. In general, a company should favor Model 1 when the desired fill rate increases. This is especially true in cases where the production capacity is high and the demand split becomes more balanced. A company should prefer Model 3, when there are low fill rates, high utilizations, and customers are of different sizes and have different demand patterns.

The use of policies such as the heuristic is possible only because of the advanced ERP systems and the gathering of real-time data. The importance of exploring such policies is recognized by Sahin and Robinson (2002) who note, “development and testing of multi-echelon inventory control procedures based on POS data capture and full information sharing is a pressing research area.” Furthermore, the authors acknowledge the need for decision rules that offer an alternative to such “best practices” as reorder point policies. Essentially, this heuristic is an alternative policy to traditional reorder point policies.
5. CONCLUSIONS AND FUTURE WORK

Two supply chain structures, a single-product serial supply chain and a multiproduct arborescent supply chain, were investigated in this dissertation. As a result of these two studies, insight into several practical issues surrounding VMI programs is gained, for instance the number of VMI customers and the sharing of VMI benefits.

The first study investigated a supply chain having a single VMI customer, where the benefit of VMI is based on improved production and shipment timing. It appears under this supply chain structure, based purely on inventory levels, that VMI can be a win-lose proposition. One participant will experience reduced inventory levels, while the other has its inventory level reduced. This leads to an important consideration for VMI contracts. Only when benefits are shared across the supply chain does VMI become beneficial for both parties. Thus, there must be a mechanism in place to enable equitable sharing of a VMI program’s benefits between the manufacturer and the customer.

There are several plausible alternatives for sharing benefits under VMI. If the supply chain had been operating under an MTS strategy prior to the move to VMI, then the manufacturer would likely obtain the most benefit from the move and thus would need to share this benefit with the customer. In this case, the most practical manner to share the benefit would be a discounted price offered to the customer. If the supply chain had previously been operating under an MTO strategy, then a move to VMI would mostly benefit the customer. Consequently, an increase in business or purchase volume would likely be the method chosen to share VMI’s benefits with the manufacturer.

Without the sharing of VMI benefits, a manufacturer’s assessment of VMI depends on the replenishment strategy VMI replaced, either MTO or MTS. If the
manufacturer originally used an MTS strategy before entering into a VMI program, then
the manufacturer could experience substantial inventory savings from the move and thus
would view VMI favorably. This appears to be the case for at least some companies,
such as Procter & Gamble (Clark and McKenney 1995). On the other hand, if a VMI
program replaces an MTO replenishment policy, then the manufacturer is likely to
experience an increase in its inventory level.

This dissertation also illustrated that VMI’s benefit is larger when a manufacturer
has multiple VMI customers. By having more VMI customers, the manufacturer has
more information with which it can schedule production and minimize such problems as
multiple customers needing their products manufactured at approximately the same time.
Thus, one possible reason why there have been reports of unsuccessful VMI
implementations is that there have not been enough participants sharing inventory
information to improve production and shipping scheduling.

In comparing the two studies of this dissertation, it is interesting to see that they
yield opposite results for the effect of utilization. When there is a single VMI customer,
increased utilization reduces VMI’s benefit, whereas its benefit increases when there are
multiple customers. Thus, from a managerial standpoint, it would seem that there is a
large incentive for a manufacturer to instigate VMI with more customers. In this case, as
indicated in the second study’s demand split factor, it appears that the best strategy is to
choose another customer having a product with very different demand characteristics.
The reason is that when there is a large disparity in size between customers, their
production needs tend to interfere more often. So by choosing a second customer in this
way, the manufacturer is better able to control this interference.
We can extend this research in several interesting areas. For the first study, we found the value of VMI relative to MTO and MTS systems depends heavily on the holding cost ratio between the manufacturer and the customer. In obtaining these results, we assumed that demand was stationary. An interesting extension to this work would be to determine the benefit of VMI when demand is nonstationary. Several demand patterns could be investigated, including increasing demand, decreasing demand, and cyclic or seasonal demand. It is expected that the value of VMI would be greater when the manufacturer has inventory information in these nonstationary demand environments.

Finally, the second study found the heuristic to offer significant benefits in environments having two customers. A worthwhile extension would be to investigate VMI’s benefit when the number of customers is even larger. Here, rather than all customers participating in the VMI program, we could manipulate the number of VMI participants while keeping the total number of customers the same. As such, the VMI manufacturer would have only partial inventory information, and some customers would continue to order in a traditional manner. Because the second study of this dissertation found VMI to be most beneficial when customers have very different demand rates, we could investigate the best strategy for a manufacturer to initiate a VMI program. For example, given a variety of customers who are not VMI participants, we could determine which customer offers the most benefit of being added first and, given this customer has been added, which customer would be best to add second, third, fourth, and so on.
APPENDICES

Appendix A. Simulation Code for Study 1

'this program simulates a serial supply chain using one of several policies
'---POLY(1) = Make to stock - first customer to place order is first priority
'---POLY(2) = Make to order - first customer to place order is first priority
'---POLY(3) = VMI - produce when customer has inventory below s1 and ship
'                   when it hits s2
'this simulation uses several lists in simlib
'---list(2) is the queue of orders for shipping
'---list(3) is the queue of orders for manufacture
'for both transfer(1) is the time and transfer(2) is the customer number
'this simulation also makes use of the master list-list(100) for event tracking
'---transfer(1) is the event time
'---transfer(2) is the event type
'---transfer(3) is the customer number
'---transfer(4) is the type of order(used only for VMI)

Option Explicit
Option Base 1
Dim LTCOUNT As Integer 'tracks leadtime factor levels
Dim UTCNT As Integer 'tracks utilization factor levels
Dim NREP As Integer 'Number of replenishments
Dim IC As Single 'Customer inventory
Dim IM As Single 'Manufacturer inventory
Dim IT As Single 'Inventory in transit
Dim ICPREV As Single 'Customer inventory from previous period
Dim IMPREV As Single 'Manufacturer inventory from previous period
Dim ITPREV As Single 'In transit inventory from previous period
Dim Q As Single 'Order Quantity
Dim LTIME As Single 'Leadtime
Dim PREVTIME As Single 'Time of last event
Dim S1 As Single 'Production signal
Dim S2 As Single 'Shipping signal
Dim PRATE As Single 'Production rate
Dim DRATE As Single 'Demand rate
Dim PTIME As Single 'Production time
Dim SIGMA As Single 'Period standard deviation
Dim PIND As Integer 'Production indicator
Dim SIND As Integer 'Shipping indicator
Dim SSEED As Long 'Start seed
Dim MAXREP As Integer 'Number of replenishments before
Dim POLY As Integer 'Policy (1=MTS, 2=MTO, 3=VMI)
Dim ICSTAT As Single 'cust inv. statistic
Dim IMSTAT As Single 'man inv. statistic
Dim ITSTAT As Single 'intransit inv statistic
Dim ENDINV As Single 'used to calculate safety stock
Dim SHORT As Single 'used to calculate fill rate
Dim ORDERFLAG As Single 'order flag
Dim PFLAG As Single 'production flag
Dim CUS As Integer 'customer number
Dim ONORDER As Single 'inv on order with customer (j)
Dim PONORDER As Single 'inv on production order with cus(j)
Dim i As Integer 'used to write outputs
Dim ECH As Single 'echelon inventory level

Sub MASTER()
Dim Invcount As Single
Invcount = 0
Application.ScreenUpdating = False
ITCOUNT = 0
POLY = 0
ENV = 0
CTYPE = 0
For LTCOUNT = 1 To 4
For UTCNT = 1 To 4
For CTYPE = 1 To 4
Invcount = 0
For ENV = 1 To 4
For POLY = 1 To 3
Call ReadInputs
Call Main
Call WriteOutputs
Next
ITCOUNT = ITCOUNT + 1
ActiveWorkbook.Save
Next
Next
Next
Next
ActiveWorkbook.Save
End Sub

Sub ReadInputs()
With Worksheets("sheet1").Range("c2")
SSEED = .Offset(1, 4)
MAXREP = .Offset(2, 4)
End With

DRATE = 100
If ENV = 1 Then
Q = 100
ElseIf ENV = 2 Then
Q = 200
ElseIf ENV = 3 Then
Q = 300
Else
Q = 400
End If

If UTCNT = 1 Then PRATE = 100 / 0.6
If UTCNT = 2 Then PRATE = 100 / 0.7
If UTCNT = 3 Then PRATE = 100 / 0.8
If UTCNT = 4 Then PRATE = 100 / 0.9

LTIME = LTCOUNT
If CTYPE = 1 Then
FRCOST = (1 - 0.7) * Q
ElseIf CTYPE = 2 Then
    FRCOST = (1 - 0.8) * Q
ElseIf CTYPE = 3 Then
    FRCOST = (1 - 0.9) * Q
ElseIf CTYPE = 4 Then
    FRCOST = (1 - 0.98) * Q
End If

PTIME = Q / PRATE
End Sub

Sub main()
    Application.ScreenUpdating = False
    Call Initialize
    Call InitializeSim
    Call GetStartSeeds(1, 1000000, SSEED)

    'run the simulation until time runs out
    Do
        Call Timing
        Call UpdateStats
        CUS = Transfer(3)
        If NextEType = 1 Then
            Call PRODSTART
        ElseIf NextEType = 2 Then
            Call ProdFin
        ElseIf NextEType = 3 Then
            Call SendShip
        ElseIf NextEType = 4 Then
            Call RecShip
        ElseIf NextEType = 5 Then
            Call Arrival
        ElseIf NextEType = 6 Then
            'Call Order
        ElseIf NextEType = 7 Then
            'Call OrderArr
        End If
        PREVTIME = Time
    Loop Until NREP >= MAXREP
    ESPRC = (SHORT) / (NREP)
    If ESPRC = 0 Then
        ADJUST = S2 / Q
        ADJUST2 = S1 / Q
    Else
        ADJUST = 0
        ADJUST2 = 0
    End If
End Sub

Sub InitializeSim()
    Dim j As Integer
ICPREV = 0
SIND = 0
ICSTAT = 0
ENDINV = 0
SHORT = 0
ORDERFLAG = 0
ONORDER = 0
PONORDER = 0
IT = 0
ICPREV = 0
ITSTAT = 0
ITPREV = 0
NREP = 0
PREVTIME = 0
PIND = 0
ECH = 0
IMPREV = 0
PFLAG = 0
IMSTAT = 0

i = 0
If POLY = 1 Then
   IC = Int(S2 + 1)
   IM = Q
   Transfer(1) = 0
   Transfer(2) = 5
   Transfer(3) = 1
   Call File(3, MAXLISTS)
ElseIf POLY = 2 Then
   IC = Int(S2 + 1)
   IM = 0
   Transfer(1) = 0
   Transfer(2) = 5
   Transfer(3) = 1
   Call File(3, MAXLISTS)
ElseIf POLY >= 3 Then
   IC = Int(S1 + 1)
   IM = 0
   Transfer(1) = 0
   Transfer(2) = 5
   Transfer(3) = 1
   Call File(3, MAXLISTS)
   End If
End Sub

Sub Arrival()
   'calculate customer and manufacturer inventory levels
   IC = IC - 1

   If POLY = 1 Then
      Call MTS
   ElseIf POLY = 2 Then
      Call MTO
   ElseIf POLY >= 3 Then
      Call VMI
   End If
End Sub
'schedule next arrival
Transfer(1) = Time + Gamma1(0.16, 1 / 16, 1)
Transfer(2) = 5
Call File(3, MAXLISTS)
End Sub

Sub PRODSTART()
'Set the production indicator to one
PIND = 1
ORDERFLAG = 0

'Schedule completion time
Transfer(1) = Time + PTIME
Transfer(2) = 2
Call File(3, MAXLISTS)
End Sub

Sub ProdFin()
'Set the production indicator to zero
PIND = 0

'raise manufacturer's inventory level to Q
IM = Q

call logic
If POLY = 1 Then
  Call MTS
ElseIf POLY = 2 Then
  Call MTO
ElseIf POLY >= 3 Then
  Call VMI
End If
End Sub

Sub SendShip()
'Set the production indicator to one
SIND = SIND + 1

decrease manufacturer's inventory by Q
IM = IM - Q
IT = IT + Q

'in order to calculate statistics correctly, the previous inventory levels
'at the manufacturer and in transit must be updated
************************
IMPREV = IM
ITPREV = IT
************************

'Schedule arrival time
Transfer(1) = Time + LTIME
Transfer(2) = 4
Call File(3, MAXLISTS)
End Sub
Sub RecShip()
    'remove one shipment from the shipment queue
    SIND = SIND - 1

    'update inventory levels
    IT = IT - Q
    ENDSINV = ENDSINV + IC

    'calculate customer inventory immediately before shipment arrives and use
    'this to calculate the ESPRC and safety stock
    If IC < 0 Then SHORT = SHORT - IC
    IC = IC + Q

    'in order to calculate statistics correctly, the previous inventory levels
    'at the manufacturer and in transit must be updated
    '*******************
    ICPREV = IC
    ITPREV = IT
    '*******************
    NREP = NREP + 1
    Time = Time
End Sub

Sub MTS()
    'nothing can be shipped while producing, so only look at situation when no production is occurring
    If PIND = 0 Then
        'if the manufacturer has no inventory, immediately begin production since make to stock
        If IM = 0 Then
            Call PRODSTART
        'since PIND=0, if the manufacturer does not have zero units, it must have Q units
        'in this case, see if IC is below reorder point
        Else
            If (IC + IT) <= S2 Then
                Call SendShip
                Call PRODSTART
            End If
        End If
    End If
End Sub

Sub MTO()
    'nothing can be shipped while producing, so only look at situation when no production is occurring
    If PIND = 0 Then
        'if the manufacturer has no inventory, see if you are below the reorder point and if so
        'start producing
        If IM = 0 Then
            If (IC + IT) <= S2 Then
                Call PRODSTART
            End If
        'if you are not producing and have >0 inventory then you must have Q units, in
        'which case you need to ship it immediately
        Else
            Call SendShip
            If (IC + IT) <= S2 Then
                Call PRODSTART
            End If
        End If
    End Sub
Sub VMI()
' nothing can be shipped while producing, so only look at situation when no production is occurring
If PIND = 0 Then
' if the manufacturer has no inventory, and you are below the production signal point, start producing
If IM = 0 Then
If (IC + IT) <= S1 Then
Call PRODSTART
End If
' if you are not producing and have >0 inventory then you must have Q units, in
' which case you need to see if you are above s2
Else
' if the manufacturer does not have inventory and you are below the reorder point,
'send the shipment
If (IC + IT) <= S2 Then
Call SendShip
End If
End If
End Sub

Sub UpdateStats()
Dim j As Integer
If IC < 0 Then
ICSTAT = ICSTAT
Else
ICSTAT = ICSTAT + (Time - PREVTIME) * IC
End If

ITSTAT = ITSTAT + (Time - PREVTIME) * IT
ECH = ECH + IC + IT

' if the mfg is producing, calculate its inventory
If PIND = 1 Then
IM = IM + (Time - PREVTIME) * PRATE
End If

IMSTAT = IMSTAT + (Time - PREVTIME) * (IM + IMPREV) / 2
IMPREV = IM

End Sub

Sub WriteOutputs()
With Worksheets("Results").Range("i3")
.Offset(0, -7) = "Policy"
.Offset(0, -6) = "Q"
.Offset(0, -5) = "PRATE"
.Offset(0, -4) = "Utilization"
.Offset(0, -3) = "LTime"
.Offset(0, -2) = "Fill Rate"
.Offset(0, -1) = "s1"
.Offset(0, 0) = "s2"
.Offset(0, 1) = "Ic"
End With
If POLY = 1 Then
    Offset(ITCOUNT * 3 + POLY, -7) = "MTS"
ElseIf POLY = 2 Then
    Offset(ITCOUNT * 3 + POLY, -7) = "MTO"
ElseIf POLY = 3 Then
    Offset(ITCOUNT * 3 + POLY, -7) = "VMI - 1"
ElseIf POLY = 4 Then
    Offset(ITCOUNT * 3 + POLY, -7) = "VMI - 2"
ElseIf POLY = 5 Then
    Offset(ITCOUNT * 3 + POLY, -7) = "VMI - 3"
ElseIf POLY = 6 Then
    Offset(ITCOUNT * 3 + POLY, -7) = "VMI - 4"
End If

Offset(ITCOUNT * 3 + POLY, -6) = Q
Offset(ITCOUNT * 3 + POLY, -5) = PRATE
Offset(ITCOUNT * 3 + POLY, -4) = DRATE / PRATE
Offset(ITCOUNT * 3 + POLY, -3) = LTIME
Offset(ITCOUNT * 3 + POLY, -2) = 1 - FRCOST / Q

If POLY >= 3 Then
    Offset(ITCOUNT * 3 + POLY, -1) = S1
Else
    Offset(ITCOUNT * 3 + POLY, -1) = 0
End If

Offset(ITCOUNT * 3 + POLY, 0) = S2
Offset(ITCOUNT * 3 + POLY, 1) = ICSTAT / Time
Offset(ITCOUNT * 3 + POLY, 2) = IMSTAT / Time
Offset(ITCOUNT * 3 + POLY, 3) = SHORT / NREP
Offset(ITCOUNT * 3 + POLY, 4) = 1 - SHORT / (Q * NREP)
Offset(ITCOUNT * 3 + POLY, 5) = ENDINV / NREP
End With
End Sub

Sub Sortresults()
    Dim i As Integer
    Dim j As Integer
    Dim k As Integer
    Dim val As Single

    With Worksheets("results").Range("p3")
        For i = 3 To 999 Step 3
            val = .Offset(i, 0)
            For j = -13 To -9
                .Offset(k, j + 18) = .Offset(i, j)
            Next
            .Offset(k, 10) = .Offset(i, 0)
            k = k + 1
        Next
    End With
End Sub
End Sub
Appendix B – Simulation Code for Study 2

Dim POLY As Integer
Dim CTIME As Single, PREVTIME As Single, MAXTIME As Long
Dim ONHAND(1 To 2) As Single, ONORDER(1 To 2) As Single
Dim NXTORDER(1 To 2, 1 To 10) As Single, NXTPROD(1 To 2) As Single
Dim ENDINV(1 To 2) As Single, AVGINV(1 To 2) As Single
Dim PRVINV(1 To 2) As Single, MFGINV(1 To 2) As Single
Dim ALPHA(1 To 2) As Single, BETA(1 To 2) As Single, MEAN(1 To 2) As Single
Dim LOT As Integer, ESPRC(1 To 2) As Single, VAR As Single
Dim Q(1 To 2) As Integer, FR As Single, CV As Single, ULN As Single, SPLIT As Integer
Dim PRATE As Single, FRCOST As Single, IDLE As Integer, PROD As Integer
Dim RPNT(1 To 2) As Single, LTIME As Integer, NREP(1 To 2) As Integer, AVGESPRC As Single
Dim ADJUST As Single, S2 As Single, S1 As Single, S2MID As Single, S2LOW As Single, S2HIGH As Single
Dim DELTA As Single, XLOW As Single, FXLOW As Single, XMID As Single, FXMID As Single
Dim XHIGH As Single, FXHIGH As Single, COUNT As Integer, STATSTART As Integer, STATIME As Single
Dim MARK(1 To 2)
Dim SQFR(1 To 2) As Double, SMFR(1 To 2) As Single, SQCus(1 To 2) As Double
Dim SMCus(1 To 2) As Single, SQMFG(1 To 2) As Double, SMMFG(1 To 2) As Single
Dim REP As Integer, MAXREP As Integer, AVGREP(1 To 2) As Single

Sub Master()
Dim j As Integer, i As Integer
Call GetStartSeeds(1, 1000, 1766)

COUNT = 1
MAXREP = 10

For j = 1 To 1024
' CODE = LOT + 10 * FR + 100 * CV + ULN * 1000 + 10000 * SPLIT
  For i = 1 To 2
    SQFR(i) = 0
    SMFR(i) = 0
    SQCus(i) = 0
    SMCus(i) = 0
    SQMFG(i) = 0
    SMMFG(i) = 0
    AVGREP(i) = 0
  Next

  Call Inputs

  For REP = 1 To MAXREP
    Call Main2
    Call DATACALC
  Next

  Call Writeout

  ActiveWorkbook.Save
  COUNT = COUNT + 1
Next
ActiveWorkbook.Save
End Sub
Sub Inputs()
Dim i As Integer
With Worksheets("data").Range("a2")
    LOT = .Offset(COUNT, 0)
    POLY = .Offset(COUNT, 1)
    S1 = .Offset(COUNT, 2)
    S2 = .Offset(COUNT, 3)
    VAR = .Offset(COUNT, 4)
    Q(1) = .Offset(COUNT, 5)
    Q(2) = .Offset(COUNT, 6)
    MEAN(1) = .Offset(COUNT, 7)
    MEAN(2) = .Offset(COUNT, 8)
    PRATE = .Offset(COUNT, 9)
    FRCOST = .Offset(COUNT, 10)
End With

If VAR = 0.5 Then
    ALPHA(1) = 4
ElseIf VAR = 0.75 Then
    ALPHA(1) = 1.77619334856808
ElseIf VAR = 1 Then
    ALPHA(1) = 1
ElseIf VAR = 1.5 Then
    ALPHA(1) = 0.444444445305522
End If

End Sub

Sub Main2()
Dim i As Integer
Call INIT

Do
    Call Period
    PREVTIME = CTIME
    CTIME = CTIME + 1
Loop Until (NREP(1) + NREP(2)) >= 2000

For i = 1 To 2
    ESPRC(i) = (ESPRC(i) / (NREP(i)))
    If ESPRC(i) = 0 Then
        ADJUST = S2 / Q(i)
    Else
        ADJUST = 0
        ADJUST2 = 0
    End If
Next

AVGESPRC = ((1 - ESPRC(1) / Q(1)) * NREP(1) + (1 - ESPRC(2) / Q(2)) * NREP(2)) / (NREP(1) + NREP(2))

If POLY = 1 Then
    AVGESPRC = 1 - ESPRC(2) / Q(2)
    ESPRC(1) = 1 - ESPRC(1) / Q(1)
End If
End Sub
Sub INIT()
Dim i As Integer, RAT As Single, j As Integer
STATIME = 0
STATSTART = 0
CTIME = 0
IDLE = 0
LTIME = 1
NREP(1) = 0
NREP(2) = 0
ADJUST = 0
For i = 1 To 2
    ONORDER(i) = 0
    ONHAND(i) = MEAN(i) * Q(i) / PRATE * i * 11
    NXTPROD(i) = -1
    PRVINV(i) = Q(i)
    ENDINV(i) = 0
    AVGINV(i) = 0
    PRVINV(i) = 0
    MFGINV(i) = 0
    PROD = 0
    For j = 1 To 10
        NXTORDER(i, j) = -1
    Next
    MARK(i) = 1
Next
PREVTIME = 0
AVGESPRC = 0
ADJUST = 0
End Sub

Sub Period()
Dim i As Integer, j As Integer
For i = 1 To 2
    If NXTPROD(i) = CTIME Then
        NXTORDER(i, MARK(i)) = CTIME + LTIME
        MARK(i) = MARK(i) + 1
        IDLE = 0
    End If
    If NXTORDER(i, 1) = CTIME Then
        If ONHAND(i) < 0 Then
            ESPRC(i) = ESPRC(i) - ONHAND(i)
        End If
        NREP(i) = NREP(i) + 1
        ONHAND(i) = ONHAND(i) + Q(i)
        ONORDER(i) = ONORDER(i) - Q(i)
        For j = 1 To MARK(i)
            NXTORDER(i, j) = NXTORDER(i, j + 1)
        Next
        MARK(i) = MARK(i) - 1
    End If
Next
If IDLE = 0 Then
    'make all decisions
    If POLY = 1 Then Call ECH
    If POLY = 2 Then Call INST
    If POLY = 3 Then Call spare4
End If

For i = 1 To 2
    'Experience demand
    If ALPHA(1) < 1 Then
        ONHAND(i) = ONHAND(i) - Gamma1(ALPHA(1), BETA(i), 1)
    Else
        ONHAND(i) = ONHAND(i) - Gamma2(ALPHA(1), BETA(i), 1)
    End If

    'calculate inventory
    If STATSTART = 0 And NREP(1) > 0 And NREP(2) > 0 Then
        STATIME = CTIME
        STATSTART = 1
        NREP(1) = 0
        NREP(2) = 0
        ESPRC(1) = 0
        ESPRC(2) = 0
    End If

    If STATSTART = 1 Then
        ENDINV(i) = ENDINV(i) + ONHAND(i)
        AVGINV(i) = AVGINV(i) + (PRVINV(i) + ONHAND(i)) / 2
        PRVINV(i) = ONHAND(i)
        If IDLE = 1 And PROD = i Then MFGINV(i) = MFGINV(i) + PRATE
        AVGREP(i) = AVGREP(i) + 1
    Else
        PRVINV(i) = ONHAND(i)
    End If
Next

End Sub

Sub ECH()
    Dim i As Integer, FLAG As Integer, j As Integer
    FLAG = 0

    If (ONHAND(1) + ONORDER(1)) <= S1 And IDLE = 0 Then FLAG = FLAG + 1
    If (ONHAND(2) + ONORDER(2)) <= S2 And IDLE = 0 Then FLAG = FLAG + 2

    If FLAG <> 0 Then
        If FLAG < 3 Then
            ONORDER(FLAG) = ONORDER(FLAG) + Q(FLAG)
            IDLE = 1
            NXTPROD(FLAG) = CTIME + (Q(FLAG) / PRATE)
            PROD = FLAG
        Else
            If (ONHAND(1) + ONORDER(1)) / MEAN(1) <= (ONHAND(2) + ONORDER(2)) / MEAN(2) Then
                ONORDER(1) = ONORDER(1) + Q(1)
                IDLE = 1
            Else
                ONORDER(2) = ONORDER(2) + Q(2)
                IDLE = 2
            End If
        End If
    End If

End Sub
Sub INST()
Dim i As Integer, FLAG As Integer
FLAG = 0
If (ONHAND(1) + ONHAND(2) + ONORDER(1) + ONORDER(2)) <= S2 And IDLE = 0 Then
    IDLE = 1
    If (ONHAND(1) + ONORDER(1)) / MEAN(1) <= (ONHAND(2) + ONORDER(2)) / MEAN(2) Then
        NXTPROD(1) = CTIME + (Q(1) / PRATE)
        PROD = 1
        ONORDER(1) = ONORDER(1) + Q(1)
    Else
        NXTPROD(2) = CTIME + (Q(2) / PRATE)
        PROD = 2
        ONORDER(2) = ONORDER(2) + Q(2)
    End If
End If
End Sub

Sub spare4()
Dim i As Integer, NEED1 As Single, NEED2 As Single, FIRST As Single, SECOND As Single
Dim ES1A As Single, ES1B As Single, ES2A As Single, ES2B As Single, RNTOTAL As Single
Dim shit1, shit2, AVG2A As Single, AVG3 As Single, AVG2B As Single, AVG3B As Single
Dim ES3A As Single, ES3B As Single, MAXA As Single, MAXB As Single
Dim FR1A As Single, FR1B As Single, FR2A As Single, FR2B As Single
FLAG = 0
NEED1 = 1
NEED2 = 2
ES1A = 1 - ESCALC(NEED1, Q(NEED1) / PRATE + 1, 1) / Q(NEED1)
ES1B = 1 - ESCALC(NEED2, Q(NEED2) / PRATE + 1, 1) / Q(NEED2)
ES2A = 1 - ESCALC(NEED1, (Q(NEED1) + Q(NEED2)) / PRATE + 1, 1) / Q(NEED1)
ES2B = 1 - ESCALC(NEED2, (Q(NEED2) + Q(NEED1)) / PRATE + 1, 1) / Q(NEED2)
'what if I made A then B or B then A?
AVG2A = (ES1A + ES2B) / 2
AVG2B = (ES1B + ES2A) / 2
MAXA = Application.WorksheetFunction.Max(AVG2A, AVG2B)
MAXB = Application.WorksheetFunction.Max(ES1A, ES1B)
'If you expect to exceed the desired fill rate at the present time, produce
If IDLE = 0 Then
    'if you need to produce one product b/c the other will stockout by the time
'you get done with the first...
If MAXA <= (FRCOST + S2) Then
    If AVG2A = MAXA Then
        NEED1 = 1
        'If ES1B < ES1A Then
        '    NEED1 = NEED1
        'End If
    ElseIf AVG2B = MAXA Then
        NEED1 = 2
        'If ES1A < ES1B Then
        '    NEED1 = NEED1
        'End If
    End If
    NXTPROD(NEED1) = CTIME + (Q(NEED1) / PRATE)
    PROD = NEED1
    ONORDER(NEED1) = ONORDER(NEED1) + Q(NEED1)
    IDLE = 1
ElseIf ES1A < FRCOST Or ES1B < FRCOST Then
    If ES1A < ES1B Then
        NEED1 = 1
        NEED2 = 2
    Else
        NEED1 = 2
        NEED2 = 1
    End If
    NXTPROD(NEED1) = CTIME + (Q(NEED1) / PRATE)
    PROD = NEED1
    ONORDER(NEED1) = ONORDER(NEED1) + Q(NEED1)
    IDLE = 1
ElseIf (1 - ESCALC(1, Q(1) / PRATE + 1, 2) / Q(1)) < FRCOST Or (1 - ESCALC(2, Q(2) / PRATE + 1, 2) / Q(2)) < FRCOST Then
    If (1 - ESCALC(1, Q(1) / PRATE + 1, 2) / Q(1)) < (1 - ESCALC(2, Q(2) / PRATE + 1, 2) / Q(2)) Then
        NEED1 = 1
        NEED2 = 2
    Else
        NEED1 = 2
        NEED2 = 1
    End If
    If ((1 - ESCALC(NEED1, (Q(NEED1) / PRATE + 1), 2) / Q(NEED1)) + (1 - ESCALC(NEED1, Q(NEED1) / PRATE + 1, 1)) / Q(NEED1))) / 2 > FRCOST Then
        NXTPROD(NEED1) = CTIME + (Q(NEED1) / PRATE)
        PROD = NEED1
        ONORDER(NEED1) = ONORDER(NEED1) + Q(NEED1)
        IDLE = 1
    End If
End If
End If
CTIME = CTIME
End Sub
Function ESCALC(cus As Single, LT As Single, OPT As Integer) As Single
Dim FIRST As Single, SECOND As Single
If OPT = 1 Then
    If ONHAND(cus) + ONORDER(cus) <= 0 Then
        ESCALC = (LT * ALPHA(1)) * BETA(cus) - (ONHAND(cus) + ONORDER(cus))
    Else
        FIRST = Application.WorksheetFunction.GammaDist(ONHAND(cus) + ONORDER(cus), LT * ALPHA(1) + 1, BETA(cus), True)
        SECOND = Application.WorksheetFunction.GammaDist(ONHAND(cus) + ONORDER(cus), LT * ALPHA(1), BETA(cus), True)
        ESCALC = LT * ALPHA(1) * BETA(cus) * (1 - FIRST) - (ONHAND(cus) + ONORDER(cus)) * (1 - SECOND)
    End If
Else
    If ONHAND(cus) + ONORDER(cus) <= 0 Then
        ESCALC = ((LT + 1) * ALPHA(1)) * BETA(cus) - (ONHAND(cus) + ONORDER(cus))
    Else
        FIRST = Application.WorksheetFunction.GammaDist(ONHAND(cus) + ONORDER(cus), (LT + 1) * ALPHA(1) + 1, BETA(cus), True)
        SECOND = Application.WorksheetFunction.GammaDist(ONHAND(cus) + ONORDER(cus), (LT + 1) * ALPHA(1), BETA(cus), True)
        ESCALC = (LT + 1) * ALPHA(1) * BETA(cus) * (1 - FIRST) - (ONHAND(cus) + ONORDER(cus)) * (1 - SECOND)
    End If
End If
End Function

Function CALC2(INV As Single, LT As Single, cus As Single) As Single
Dim FIRST As Single, SECOND As Single
If INV <= 0 Then
    CALC2 = (LT * ALPHA(1)) * BETA(cus) - (INV)
Else
    FIRST = Application.WorksheetFunction.GammaDist(INV, LT * ALPHA(1) + 1, BETA(cus), True)
    SECOND = Application.WorksheetFunction.GammaDist(INV, LT * ALPHA(1), BETA(cus), True)
    CALC2 = LT * ALPHA(1) * BETA(cus) * (1 - FIRST) - (INV) * (1 - SECOND)
End If
End Function

Sub DATACALC()
'need to calc data for esprc1,esprc2,mfginv,cus1inv,cus2inv
'one variable to sum the squares the result and the other to sum
If POLY > 1 Then
    For i = 1 To 2
        SQFR(i) = SQFR(i) + (1 - (ESPRC(i) / Q(i))) ^ 2
        SMFR(i) = SMFR(i) + (1 - (ESPRC(i) / Q(i)))
    Next
Else
    SQFR(1) = SQFR(1) + ESPRC(1) ^ 2
    SMFR(1) = SMFR(1) + ESPRC(1)
    SQFR(2) = SQFR(2) + AVGESPRC ^ 2
    SMFR(2) = SMFR(2) + AVGESPRC
End If

For i = 1 To 2
    AVGREP(i) = AVGREP(i) + NREP(i)
End For
SQCu(i) = SQCu(i) + ((ENDINV(i) / (CTIME - STATIME)) ^ 2
SMCu(i) = SMCu(i) + ((ENDINV(i) / (CTIME - STATIME))
SQMFG(i) = SQMFG(i) + ((MFGINV(i) / (CTIME - STATIME)) ^ 2
SMMFG(i) = SMMFG(i) + ((MFGINV(i) / (CTIME - STATIME))
Next
End Sub

Sub Writeout()
AVGREP(1) = AVGREP(1) / MAXREP
AVGREP(2) = AVGREP(2) / MAXREP
With Worksheets("results").Range("f2")
. Offset(0, -5) = "Lot"
. Offset(0, -4) = "POLY"
. Offset(0, -3) = "S1"
. Offset(0, -2) = "S2"
. Offset(0, -1) = "VAR"
. Offset(0, 0) = "Q1"
. Offset(0, 1) = "Q2"
. Offset(0, 2) = "MEAN(1)"
. Offset(0, 3) = "MEAN(2)"
. Offset(0, 4) = "PRATE"
. Offset(0, 5) = "FRCOST"
. Offset(0, 6) = "Cus 1 Avg"
. Offset(0, 7) = "Cus 1 Std"
. Offset(0, 8) = "Cus 2 Avg"
. Offset(0, 9) = "Cus 2 Std"
. Offset(0, 10) = "Mfg 1 Avg"
. Offset(0, 11) = "Mfg 1 Std"
. Offset(0, 12) = "Mfg 2 Avg"
. Offset(0, 13) = "Mfg 2 Std"
. Offset(0, 14) = "Cus 1 FR"
. Offset(0, 15) = "Cus 1 FRstd"
. Offset(0, 16) = "Cus 2 FR"
. Offset(0, 17) = "Cus 2 FRstd"
. Offset(0, 18) = "Avg Cus Inv"
. Offset(0, 19) = "Avg FR"
. Offset(COUNT, -5) = LOT
. Offset(COUNT, -4) = POLY
If POLY = 1 Then . Offset(COUNT, -3) = S1
. Offset(COUNT, -2) = S2
. Offset(COUNT, -1) = VAR
. Offset(COUNT, 0) = Q(1)
. Offset(COUNT, 1) = Q(2)
. Offset(COUNT, 2) = MEAN(1)
. Offset(COUNT, 3) = MEAN(2)
. Offset(COUNT, 4) = PRATE
. Offset(COUNT, 5) = FRCOST
. Offset(COUNT, 6) = SMCus(1) / MAXREP
. Offset(COUNT, 7) = ((SQCu(1) - (SMCu(1) ^ 2) / MAXREP) / (MAXREP - 1)) ^ 0.5
. Offset(COUNT, 8) = SMCus(2) / MAXREP
. Offset(COUNT, 9) = ((SQCu(2) - (SMCu(2) ^ 2) / MAXREP) / (MAXREP - 1)) ^ 0.5
. Offset(COUNT, 10) = SMMFG(1) / MAXREP
. Offset(COUNT, 11) = ((SQMFG(1) - (SMMFG(1) ^ 2) / MAXREP) / (MAXREP - 1)) ^ 0.5
. Offset(COUNT, 12) = SMMFG(2) / MAXREP
131
.Offset(COUNT, 13) = ((SQMFG(2) - (SMMFG(2) ^ 2) / MAXREP) / (MAXREP - 1)) ^ 0.5
.Offset(COUNT, 14) = SMFR(1) / MAXREP
.Offset(COUNT, 15) = ((SQFR(1) - (SMFR(1) ^ 2) / MAXREP) / (MAXREP - 1)) ^ 0.5
.Offset(COUNT, 16) = SMFR(2) / MAXREP

If ((SQFR(2) - (SMFR(2) ^ 2) / MAXREP)) < 0 Then
  .Offset(COUNT, 17) = 0
Else
  .Offset(COUNT, 17) = ((SQFR(2) - (SMFR(2) ^ 2) / MAXREP) / (MAXREP - 1)) ^ 0.5
End If
.Offset(COUNT, 18) = SMCus(1) + SMCus(2)
.Offset(COUNT, 19) = ((SMFR(1) * AVGREP(1) + SMFR(2) * AVGREP(2)) / (MAXREP * (AVGREP(1) + AVGREP(2))))
End With
End Sub
REFERENCES


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